Bargaining set with endogenous leaders: a convergence result

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Abstract. We provide a notion of bargaining set for finite economies, where endogenous proponents of objections are required to be convincing leaders in order to block allocations justifiably. We show its convergence to the set of Walrasian allocations when the economy is replicated.

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1 Introduction

The core of an economy is defined as the set of allocations which cannot be blocked or objected by any coalition. Thus, the veto mechanism that defines the core does not take into account that other agents in the economy may react to an objection and propose an alternative or counterobjection.

This two-step conception of the veto mechanism was considered by Aumann and Maschler (1964), who introduced the concept of bargaining set, containing the core of a cooperative game.[1] In the Aumann and Maschler (1964) and Davis and Maschler (1963) definitions, the original objection is proposed by a "leader" that must be excluded in any counterobjecting coalition.

Geanakoplos (1978) considered sequences of transferable utility (TU) exchange economies with smooth preferences and modified the Aumann-Davis-Maschler definition so that the "leader" was a group of agents containing a fixed (but small) fraction of the number of agents in the economy; thus, as the number of agents grew along the sequence of economies, the number of individuals in the "leader" grew proportionately. By using nonstandard analysis, he showed that this Geanakoplos bargaining set becomes asymptotically competitive as the number of agents grows. Shapley and Shubik (1984) showed that the Aumann-Davis-Maschler bargaining set is approximately competitive in replica sequences of TU exchange economies with smooth preferences. Anderson (1998) extended both Geanakoplos' result to nontransferable utility (NTU) exchange economies without smooth preferences and the Shapley and Shubik result to non-replica sequences of NTU exchange economies with smooth preferences.

On the other hand, Mas-Colell (1989) considered (NTU) economies with a continuum of agents and proposed a modification of the Aumann and Maschler bargaining set that does not involve the concept of a leader. Under conditions of generality similar to Aumann's (1964) core equivalence theorem, he showed that his bargaining set and the set of Walrasian allocations coincide. However, in contrast to Debreu and Scarf's (1963) core-convergence result, Anderson *et al.* (1997) showed the Mas-Colell (1989) and Zhou (1994) bargaining sets need not converge in replica sequences of economies, no matter how nice the preferences may be.

The presence of a "leader" that proposes the objection and precommits not to

[1] Maschler (1976) discussed the advantages that the bargaining set has over the core.

participate in any counterobjection makes it easier to create a justified objection. Indeed, it is remarkable that the designation of a leader makes a profound difference at a conceptual level in the resulting bargaining set and specially regarding convergence properties.

In this paper, we provide a notion of bargaining set where the potential proponents of the objections are determined endogenously and are required to be credible or convincing leaders in order to prevent an allocation to belong to the bargaining set. This implies that our solution differs from the previous one considered in the related literature with respect to several formal and economical aspects. It marks a difference the condition of credibility to become a leader that is endogenously determined in order to be able to propose and defend an objection in a justified way. It is also a crucial difference the fact that in our notion, if an agent belongs to an objecting coalition then any other individual of the same type involved in a counterobjection is required to improve with respect to the bundle that receives her homologue in the objection. The two-step procedure of the veto mechanism becomes particularly relevant when individuals of the same type are involved objection and counterobjection process at the same time. Our approach represents a scenario where individuals of the same type pursue common objectives as representatives of some kind of institution or organization and, in particular, it entails a specific economic interpretation for sequences of replicated economies.

Moreover, the notion of endogenous convincing leader in the objection process allows us to show that the corresponding bargaining set converges to the set of Walrasian allocations when the economy is replicated or, in other words, an allocation is Walrasian if and only if cannot be blocked by a justified objection proposed by a convincing leader in any replicated economy.

In a companion paper Hervés-Estévez and Moreno-García (2016) obtained a convergence theorem for a bargaining set without any consideration of leader, but under a necessary assumption of continuity of the Walrasian equilibrium correspondence instead. We show that the presence of endogenous leaders leads us to a limit result with no continuity condition on the equilibrium correspondence.

The rest of the work is structured as follows. In Section 2, we collect notations and preliminaries. In Section 3, we state the notion of justified objections with endogenous and credible leaders that is considered to define our leader bargaining set. In Section 4, we obtain our limit result. Finally, Section 5 contains some concluding remarks.

2 Preliminaries, notations and some previous results

Let $\mathcal{E}$ be an exchange economy with a finite set of agents $N = \{1, \ldots, n\}$, who trade a finite number m of commodities. Each consumer i has a preference relation $\succsim_i$ on the set of consumption bundles $\mathbb{R}^m_+$, with the properties of continuity, convexity[2] and strict monotonicity. Let $\omega_i \in \mathbb{R}^m_{++}$ denote the endowments of consumer i. So the economy is $\mathcal{E} = (\mathbb{R}^m_+, \succsim_i, \omega_i, i \in N)$.

An allocation x is a consumption bundle $x_i \in \mathbb{R}^m_+$ for each agent $i \in N$. The allocation x is feasible in the economy $\mathcal{E}$ if $\sum_{i=1}^n x_i \leq \sum_{i=1}^n \omega_i$. A price system is an element of the $(m-1)$-dimensional simplex of $\mathbb{R}^m_+$. A Walrasian equilibrium for $\mathcal{E}$ is a pair (p, x), where p is a price system and x is a feasible allocation such that, for every agent i, the bundle x_i maximizes her preference relation $\succeq_i$ in the budget set $B_i(p) = \{y \in \mathbb{R}^m_+ \text{ such that } p \cdot y \leq p \cdot \omega_i\}$. We denote by $W(\mathcal{E})$ the set of Walrasian allocations for the economy $\mathcal{E}$.

A coalition is a non-empty set of consumers. An allocation y is said to be attainable or feasible for the coalition S if $\sum_{i \in S} y_i \leq \sum_{i \in S} \omega_i$. Let $x \in \mathbb{R}^{mn}_+$ be a feasible allocation in the economy $\mathcal{E}$. The coalition S blocks x if there exists an allocation y which is attainable for S, such that $y_i \succsim_i x_i$ for every $i \in S$ and $y_j \succ_j x_j$ for some member j in S. When S blocks x via y we say that (S, y) is an objection to x. A feasible allocation is efficient if it is not blocked by the grand coalition, formed by all the agents. The core of the economy $\mathcal{E}$, denoted by $C(\mathcal{E})$, is the set of feasible allocations which are not blocked or objected by any coalition of agents.

It is known that, under the hypotheses above, the economy $\mathcal{E}$ has Walrasian equilibrium and that any Walrasian allocation belongs to the core (in particular, it is efficient).

In this paper, we will use the fact that, regarding Walrasian equilibria, a finite

[2] The convexity of preferences we require is the following: If a consumption bundle z is strictly preferred to $\hat{z}$ so is the convex combination $\lambda z + (1-\lambda)\hat{z}$ for any $\lambda \in (0, 1)$. This convexity property is weaker than strict convexity and it holds, for instance, when the utility functions $U_i, i \in N$, representing the preferences relations $\succsim_i, i \in N$ are concave.

economy $\mathcal{E}$ with n consumers can be associated to a continuum economy $\mathcal{E}_c$ with n-types of agents as we specify next. The set of agents in the atomless economy $\mathcal{E}_c$ is $I = [0,1] = \bigcup_{i=1}^{n} I_i$, with $I_i = \left[\frac{i-1}{n}, \frac{i}{n}\right)$ if $i \neq n$; $I_n = \left[\frac{n-1}{n}, 1\right]$. All the agents in the subinterval I_i are of the same type i, that is, every agent $t \in I_i$ has preferences $\succsim_t = \succsim_i$ and endowments $\omega(t) = \omega_i$. In this case, $x = (x_1, \ldots, x_n)$ is a Walrasian allocation in $\mathcal{E}$ if and only if the step function f_x (defined by $f_x(t) = x_i$ for every $t \in I_i$) is a competitive allocation in $\mathcal{E}_c$.

Addressing continuum economies, Mas-Colell (1989) provided a notion of bargaining set and show its coincidence with the competitive allocations. The Mas-Colell's bargaining set, that we denote by B_{MC}, contains all the feasible allocations of the economy which, if objected, they could also be counterobjected.

We aim to analyze convergence properties of bargaining sets for arbitrarily large finite economies. For it, we consider that each of the n agents of the finite economy $\mathcal{E}$ behaves as a representative of a large enough number of identical individuals. Following this way of enlarging asymptotically the economy, a typical coalition $\hat{S}$ is formed by r_i members identical to each agent i in a non-empty subset of $\{1, \ldots, n\}$. Given $\hat{S}$, let $S = \{i \in N : r_i > 0\}$. Thus, $\hat{S} = (r_i, i \in S)$ is actually a coalition in any r-replica of the original n-agents economy, for every $r \geq \max\{r_i, i \in S\}$.

We remind that for each positive integer r, the r-fold replica economy $r\mathcal{E}$ of $\mathcal{E}$ is a new economy with rn agents indexed by ij, $j = 1, \ldots, r$, such that each consumer ij has a preference relation $\succsim_{ij} = \succsim_i$ and endowments $\omega_{ij} = \omega_i$. Given a feasible allocation x in $\mathcal{E}$ let rx denote the corresponding equal treatment allocation in $r\mathcal{E}$, which is given by $rx_{ij} = x_i$ for every $j \in \{1, \ldots, r\}$ and $i \in N$.

The notions that Mas-Colell (1989) provided for atomless economies can be straightforwardly translated to the replicated economy $r\mathcal{E}$ as follows:

Let x be an allocation in $\mathcal{E}$. An objection to the allocation rx is defined by a coalition $\hat{S} = (r_i, \in S)$ in $r\mathcal{E}$ and consumption bundles y_{ij}, with $i \in S$ and $j \in \{1, \ldots, r_i\}$, such that (i) $\sum_{ij \in \hat{S}} y_{ij} \leqslant \sum_{i \in S} r_i \omega_i$ and (ii) $y_{ij} \succsim_i x_i$, for every $ij \in \hat{S}$ and $y_{ij} \succ_i x_i$ for some $ij \in \hat{S}$. A counterobjection to $(\hat{S}, y)$ is defined by a coalition $\hat{T} = (a_i, i \in T)$ in $r\mathcal{E}$ and consumption plans z_{ij}, with $i \in T$ and $j \in \{1, \ldots, a_i\}$, such that (i) $\sum_{ij \in \hat{T}} z_{ij} \leqslant \sum_{i \in T} a_i \omega_i$, (ii) $z_{ij} \succ_j y_{ij}$, if consumer $ij \in \hat{T} \cap \hat{S}$ and $z_{ij} \succ_j x_j$ if $ij \in \hat{T} \setminus \hat{S}$.

An objection is justified if it is is not counterobjected by any coalition. We say that x belongs to $B_{MC}(r\mathcal{E})$ if rx has no justified objection. It is easy to check that every justified objection $(\hat{S}, y)$ to rx in $r\mathcal{E}$ belongs to the core of the economy restricted to $\hat{S}$ and then, under convexity of preferences, we have $y_{ij} \sim_i y_{ik}$ for every $ij, ik \in \hat{S}$. Thus, without loss of generality we can also consider $z_{ij} = z_i$ in the counterobjection system.

In spite of the aforementioned equivalence result obtained by Mas-Colell (1989) and in contrast to Debreu-Scarf ' s (1963) core convergence theorem, Anderson *et al.* (1997) showed that the sequence of bargaining sets $(B_{MC}(r\mathcal{E}))_{r\in\mathbb{N}}$ does not shrink to the set of Walrasian allocations, no matter how nice the preferences may be.

3 Justified objections with endogenous leaders

The concept of bargaining set depends on how justified objections are stated. Indeed, since the original definition by Aumann and Maschler (1964), a variety of different notions have been subsequently proposed. The concept of bargaining set by Mas-Colell (1989) imposes no restriction on the members that may belong to an objecting or counterobjecting coalition. However, this is not the case for the original definition of bargaining set introduced by Aumann and Maschler (1964) and Davis and Maschler (1963), neither for most of the papers that obtain convergence results for bargaining sets (Geanakoplos, 1978; Shapley and Shubik, 1984 and Anderson, 1998). In these works, it is required either the presence of an exogenous leader or a group of leaders who acts as proponent of the original objection. The underlying argument is that when an objection is proposed by a leader, any counterobjecting coalition must exclude this leader.

In which follows, we provide a different notion of bargaining set with endogenous leader as credible proposer and defender of an objection. Let x be a feasible allocation in the economy $\mathcal{E}$ and consider an equal-treatment objection to the allocation rx in $r\mathcal{E}$. That is, there are $\hat{S} = (r_i, \in S)$ and $y = (y_i, \in S)$, with $S \subseteq \{1, \ldots, n\}$ and $r_i \leq r$, such that $\sum_{i\in S} r_i y_i \leq \sum_{i\in S} r_i \omega_i$ and $y_i \succsim_i x_i$ for every $i \in S$, with strict preference for some $j \in S$. We remark that without loss of generality we assume $r_h = r$ for some $h \in S$. Note that otherwise we can consider the objection $(\hat{S}, y)$ in the replicated economy $\bar{r}\mathcal{E}$ with $\bar{r} = \max_{i\in S} r_i$. Let $\mathcal{L}_{\hat{S}} = \{i \in S : r_i = r\}$. In our framework, only a type in $\mathcal{L}_{\hat{S}}$ can behave as proponent of the objection $(\hat{S}, y)$. That is, any type that is fully represented in the blocking system may become a proposer who takes the role of a leader that supports the corresponding objection. Thus, the potential leaders are endogenously determined by the objecting procedure. Note that when we refer to the original economy $\mathcal{E}$, i.e. $r = 1$, for any objection (S, y) we have $\mathcal{L}_S = S$ and every member in the coalition may be candidate to become a leader. We stress that a plausible leader, besides being endogenously determined, has a measure that is maintained when the economy is replicated and is given by 1/n, which is not the case for other related approaches.

Given the objection $(\hat{S}, y)$ we refer to a potential leader $\ell \in \mathcal{L}_{\hat{S}}$ as convincing proponent if $(\hat{S}, y)$ cannot be counterobjected by a coalition with no member of type ℓ. To be precise, we state the following definitions.

Definition (*Counterobjection*). An objection $(\hat{S}, y) = (r_i, y_i, i \in S)$ to rx in the economy $r\mathcal{E}$ is counterobjected if there exist $(\hat{T}, z) = (\hat{r}_i, z_i, i \in T)$, with $\hat{r}_i \leq r$ for every $i \in T$, such that

(i) $\sum_{i \in \mathcal{T}} \hat{r}_i z_i \leq \sum_{i \in \mathcal{T}} \hat{r}_i \omega_i$ and

(ii) $z_i \succ_i y_i$ for every $i \in T \cap S$ and $z_i \succ_i x_i$ for every $i \in T \setminus S$.

Observe that the counterobjection mechanism we propose is different from the standard one used in Mas-Colell's approach. Indeed, in our definition if one agent participates in an objecting coalition, any other agent of the same type is not allowed to participate in a counterobjection unless she gets a more preferred bundle than the bundle her homologue obtains in the objection. This mechanism is specially relevant when individuals of the same type must behave coordinately pursuing common interests.

Definition (*Endogenous convincing leader*). The objection $(\hat{S}, y)$ to rx in the economy $r\mathcal{E}$ has a convincing leader $\ell \in \mathcal{L}_{\hat{S}}$ if there is no coalition $\hat{T} = (\hat{r}_i, i \in T)$ able to counterobjects $(\hat{S}, y)$ such that ℓ does not belong to T.

Definition (*$\mathcal{L}$-justified objection*). An objection $(\hat{S}, y) = (r_i, y_i, i \in S)$ to rx in the economy $r\mathcal{E}$ is $\mathcal{L}$-justified if it has a convincing leader.

In other words, an objection has a credible leader if there exists a type that is fully represented in the objection and such that any counterobjection requires the participation of such a type. If it is the case, we say that there is a convincing leader and the corresponding objection is $\mathcal{L}$-justified.

Definition (*Leader bargaining set $B_{\mathcal{L}}$*). We say that the feasible allocation x belongs to the leader bargaining set of $r\mathcal{E}$ and we write $x \in B_{\mathcal{L}}(r\mathcal{E})$ if the allocation rx has no $\mathcal{L}$-justified objection. That is x is not in $B_{\mathcal{L}}(r\mathcal{E})$ if rx has an objection in $r\mathcal{E}$ with a convincing leader.

We must remember that, for every r, the set of Walrasian allocations of the economy $\mathcal{E}$ is contained in the core of $r\mathcal{E}$ which is contained in $B_{\mathcal{L}}(r\mathcal{E})$.

4 A convergence result

As we have already remarked, the designation of a leader in the objecting mechanism makes a profound difference in the resulting bargaining sets, especially when the economy is enlarged with the aim of studying convergence properties. Indeed, most of the bargaining set convergence results that have been obtained depend crucially on the presence of a leader or a group of leaders (Geanakoplos; 1978, Shapley and Shubik, 1984 and Anderson, 1998). Roughly speaking, the aforementioned asymptotic results show that different notions of bargaining set involving the presence of a leader can approximately be decentralized by prices for large economies.

The notion we have provided in the previous section, specifies endogenous leaders as convincing or credible proposers and defenders of an objection and allows us to show that when we replicate the economy, the resulting bargaining set converges to the set of Walrasian allocations, in a similar way as Debreu-Scarf's limit theorem for the core, without any additional continuity property of the equilibrium correspondence as it has been required for the convergence result by Hervés-Estévez and Moreno-García (2016).

Theorem 4.1 *The allocation x is Walrasian in the economy $\mathcal{E}$ if and only if x belongs to the leader bargaining set of every replicated economy. That is,*

$$\bigcap_{r\in\mathbb{N}} B_{\mathcal{L}}(r\mathcal{E}) = W(\mathcal{E}).$$

Proof. Since $W(\mathcal{E}) \subseteq C(r\mathcal{E}) \subseteq B_{\mathcal{L}}(r\mathcal{E})$, it is immediate that $W(\mathcal{E}) \subseteq \bigcap_{r\in\mathbb{N}} B_{\mathcal{L}}(r\mathcal{E})$.

To show the converse, we define an auxiliary larger bargaining set $B^*_{\mathcal{L}}(r\mathcal{E})$, by considering counterobjections in any replicated economy. That is, the objection $(\hat{S}, y)$ to rx in the economy $r\mathcal{E}$ is $\mathcal{L}^*$-justified if there is a leader $\ell \in \mathcal{L}_{\hat{S}}$ such that there is no coalition $\bar{T} = (\bar{r}_i, i \in T)$ able to counterobjects $(\hat{S}, y)$ such that $\ell \notin T$, in any further replicated economy $\bar{r}\mathcal{E}$ with $\bar{r} \geq r$. Note that $B_{\mathcal{L}}(r\mathcal{E}) \subseteq B^*_{\mathcal{L}}(r\mathcal{E})$ and then it is enough to show that $W(\mathcal{E}) \subset B^*_{\mathcal{L}}(r\mathcal{E})$ for every replicated economy

$r\mathcal{E}$. For it, assume $x \in \bigcap_{r\in\mathbb{N}} B^*_{\mathcal{L}}(r\mathcal{E})$ and assume that x is not a Walrasian allocation in the economy $\mathcal{E}$. Let us consider the corresponding step function f_x in the associated continuum economy $\mathcal{E}_c$. We have that f_x does not belong to $B_{MC}(\mathcal{E}_c)$. Then, there exists a justified objection to f_x following Mas-Colell's definition in $\mathcal{E}_c$. By convexity of preferences, Remark 5 in Mas-Colell (1989) allows us to ensure that there is a justified objection to x that is given by (S, y) and parameters $\alpha_i, i \in S$, such that $\sum_{i\in S} \alpha_i y_i \leq \sum_{i\in S} \alpha_i \omega_i,\ y_i \succsim_i x_i$ for every $i \in S$ and $y_j \succ_j x_j$ for some $j \in S$. Moreover, $\alpha_j = 1$ and $y_i \sim_i x_i$ for every i such that $\alpha_i < 1$.

If $S = \{j\}$ the pair $(\{j\}, y_j)$ defines an objection in every replicated economy. Then, for every $r\mathcal{E}$ there is a collection T of types which excludes j and an allocation z such that (T, z) counterobjects $(\{j\}, y_j)$. Then we can find a counterobjection in $\mathcal{E}_c$ to the justified objection, which is a contradiction.

Now consider that S contains not only the type j. By continuity of preferences, we can take ε such that $(1-\varepsilon)y_j \succ_j x_j$. Let $\alpha = \sum_{\substack{i\in S\\ i\neq j}} \alpha_i$ and define the allocation $\tilde{y}$ as follows:

$$\tilde{y}_i = \begin{cases} (1-\varepsilon)y_i & \text{if } i = j \\ y_i + \frac{\varepsilon y_j}{\alpha} & \text{if } i \neq j \end{cases}$$

By construction, $\sum_{i\in S} \alpha_i \tilde{y}_i \leq \sum_{i\in S} \alpha_i \omega_i$. Since preferences are monotone $\tilde{y}_i \succ_i x_i$ for every $i \in S$. Actually, $\tilde{y}_i \succ_i y_i \succsim_i x_i$, for every $i \neq j$.

For every natural $k \in \mathbb{N}$, let α_i^k, $i \in S$ be the smallest integer greater than or equal to $k\alpha_i$. Let us denote $y_i^k = \frac{k\alpha_i}{\alpha_i^k}(\tilde{y}_i - \omega_i) + \omega_i$. Note that y_i^k converges to $\tilde{y}_i$ for every $i \in S$ and then, by continuity of preferences, we have that $y_i^k \succ_i x_i$ for every $i \in S$ and for all k large enough. In addition, $y_i^k \succ_i y_i \succsim_i x_i$ for every $i \neq j$ and for all k large enough. We remark that $y_j^k = (1-\varepsilon)y_j$ and $\alpha_j^k = 1$ for every k.

Then, the coalition with α_i^k agents of type $i \neq j$ with $i \in S$, and k agents of type j, blocks x via y^k in the replicated economy $k\mathcal{E}$. Therefore, there exists a counterobjection (T, z) to the objection (S, y^k) in some replicated economy $r\mathcal{E}$ with $r \geq k$, such that $j \notin T$. Thus, for every $i \in T$, there exists a natural number $\beta_i \leq r$, such that $\sum_{i\in T} \beta_i z_i \leq \sum_{i\in T} \beta_i \omega_i,\ z_i \succ_i y_i^k \succ_i y_i$ for every $i \in T \cap S$ and $z_i \succ_i x_i$ for every $i \in T \setminus S$. This is a contradiction with the fact that the objection (S, y) defines a justified objection to f_x in the associated continuum economy.

Q.E.D.

5 Final Remarks

Our convergence theorem shows that the presence (or absence) of a group of leaders makes a fundamental difference for the asymptotic analysis of bargaining sets.

The notion of the bargaining set with leader we state differs from those which have been considered in the related literature and, in turn, neither our convergence result can be deduced from the previous ones nor vice versa. Moreover, we show that the intersection of the bargaining sets of the sequence of the replicated economies coincides with the set of Walrasian allocations, providing an extension of the Debreu-Scarf core-convergence to bargaining sets which is not the case of the already obtained asymptotic theorems that show a convergence in measure (Anderson, 1998). We also remark that Anderson's convergence result applies to quite general sequences of finite exchange economies while ours is restricted to replica sequences of economies.

To summarize, our main contribution is a bargaining set convergence result and also a characterization of the Walrasian allocations for finite economies. That is, we show that any non-Walrasian allocation is blocked via a justified objection that is proposed by a convincing leader if the economy is large enough. Equivalently, our result provides also a further formalization of Edgeworth's conjecture, strengthening Debreu-Scarf's limit theorem on the core, by stating that every Walrasian allocation is characterized by belonging to our leader bargaining set for every replicated economy.

Finally, we point out that the consideration of endogenous and convincing leaders in the objecting system becomes the main reason that allows us to show our limit result on bargaining sets.

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