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Anisotropy can imply exponential decay in micropolar elasticity

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ABSTRACT

In this note, two problems arisen in micropolar elasticity are considered from the analytical point of view. Following the Kelvin–Voigt theory of micropolar viscoelasticity, two dissipative mechanisms are imposed: in the first problem, it is defined on the microscopic structure and, for the second problem, on the macroscopic structure. Then, an existence and uniqueness result, as well as an exponential energy decay, are proved for the first problem. Since similar arguments can be used for the second problem, only the main key points are commented.

1. Introduction

The study of the decay of solutions to problems involving elastic materials has been an objective considered in many publications. Usually, we can find a conservative mechanism (for instance, the displacement field) coupled with a dissipative mechanism. The natural question which can be posed is to know when the coupling is strong enough to ensure that the dissipation brings the whole system to the energy decay, particularly in an exponential way. Maybe, the first contribution is this sense was obtained by Dafermos [1]. In fact, he proved the decay to the equilibrium state in the case of the one-dimensional thermoelasticity and the impossibility that it could happen in the general case for higher dimensions. Later, Muñoz-Rivera [2] and Slemrod [3] showed that this decay was of exponential type for dimension one. However, we could ask ourselves about an alternative question. That is, how many dissipative mechanisms are needed to obtain the exponential energy decay in a multi-dimensional setting? In some recent papers, we have proved that the square of the dimension of the space of the dissipative mechanisms is a number higher enough to guarantee such objective [4–6]. This number can be reduced when we impose certain conditions (see [7]). In the case that we consider strain gradient elasticity, it can be less or equal to the dimension of that space [8]. It is worth noting the anisotropy condition and/or chirality to lead to this type of behavior.

Another problem involving elastic materials which has received much attention over the last years is the porous elasticity. In this case, we have a situation more unfavorable since, in the general case, we cannot expect the exponential energy decay even for the one-dimensional setting. Two dissipation mechanisms are required, but chosen in a particular way [9], unless we consider heat conduction theories of type II and III in the sense of Green and Naghdi [10–13]. In those cases, we have more coupling mechanisms which allow to obtain the exponential energy decay with a unique dissipative mechanism. Recently, we have seen that, under some assumptions of chirality, we can find the exponential energy decay assuming a number of dissipative mechanisms equals to the dimension of the space [14].

Anyway, the theory of micropolar elasticity has received big attention. This type of materials was proposed by Cosserat brothers at the beginning of the twentieth century [15], and it was studied extensively since 1960. We can recall the works of Eringen [16] and Ieşan [17] in this sense. The basic idea of these materials is that each point has six degrees of freedom. Three of them correspond to the macroscopic deformation and the other three refer to the microscopic level, where the material points admit a rotation movement. A few of recent contributions on time decay for these materials are [18–21]. We must point out that, in the case of isotropic materials, this coupling is weak and we cannot expect the exponential decay whenever we impose dissipation at both macroscopic or microscopic levels. In this note, we follow the work of Eringen [22], but introducing two dissipative terms which were suggested by the Kelvin-Voigt theory of micropolar viscoelasticity, and we aim to show that, for certain anisotropic materials, such dissipation at a macroscopic or microscopic level can be enough to bring the whole system to the equilibrium state in an exponential way. We recall that, in the case that the viscosity is present at both levels, the decay is of exponential type for isotropic materials [21], and the analysis can be extended without difficulty to the anisotropic case.

The plan of this note is the following. In the next section, the basic equations and the two problems involving micropolar materials are described. The first one includes a dissipative mechanism in the microscopic structure, meanwhile the dissipative mechanism in the

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macroscopic structure is assumed for the second one. The first problem is analyzed in Section 3, proving the existence of a unique solution and the exponential energy decay. The second problem is studied in Section 4. Since the arguments are similar to those used in the previous section, only few comments are provided in order to avoid the repetitions.

2. Field equations

The aim of this section is to propose the system of equations, the boundary conditions and the initial conditions which determine the problems that we are going to study in this paper.

The evolution equations within the linear theory of micropolar elastic materials can be written as

$$\rho\ddot{u}_i = t_{ji,j},\qquad I\ddot{\varphi}_i = m_{ji,j} + \varepsilon_{irs}t_{rs}. \tag{1}$$

In this system, ρ is the mass density, u_i is the displacement vector, t_{ij} is the stress tensor, m_{ij} is the couple stress tensor, φ_i is the micro-rotation, I is the inertia coefficient[1] and ε_{irs} is the alternating tensor.

We consider system (1) defined in a bounded domain $B \subset \mathbb{R}^3$ with a boundary assumed smooth enough to apply the divergence theorem.

We will study now the field equations associated to two different problems when system (1), and adequate boundary and initial conditions, are satisfied. First, in the case where we assume that the dissipation is produced in the microscopic structure, the constitutive equations are

$$t_{ij} = A_{ijrs}e_{rs} + B_{ijrs}\kappa_{rs},\qquad m_{ij} = B_{rsij}e_{rs} + C_{ijrs}\kappa_{rs} + C^*_{ijrs}\dot{\kappa}_{rs}. \tag{2}$$

It is convenient to recall that

$$e_{ij} = u_{j,i} + \varepsilon_{jik}\varphi_k,\quad \kappa_{ij} = \varphi_{j,i}. \tag{3}$$

It is also well known that

$$A_{ijrs} = A_{rsij},\quad C_{ijrs} = C_{rsij},\quad C^*_{ijrs} = C^*_{rsij}. \tag{4}$$

Secondly, in the case where we assume that the dissipation is produced in the macroscopic structure, the constitutive equations are now written as

$$t_{ij} = A_{ijrs}e_{rs} + B_{ijrs}\kappa_{rs} + A^*_{ijrs}\dot{u}_{r,s},\qquad m_{ij} = B_{rsij}e_{rs} + C_{ijrs}\kappa_{rs}. \tag{5}$$

In this second case, we assume that

$$A_{ijrs} = A_{rsij},\quad C_{ijrs} = C_{rsij},\quad A^*_{ijrs} = A^*_{rsij}. \tag{6}$$

If we substitute the constitutive equations (2) into the evolution equations (1), then we obtain the following system:

$$\begin{aligned}\rho\ddot{u}_i &= \left(A_{jirs}e_{rs} + B_{jirs}\kappa_{rs}\right)_{,j},\\ I\ddot{\varphi}_i &= \left(B_{rsji}e_{rs} + C_{jirs}\kappa_{rs} + C^*_{jirs}\dot{\kappa}_{rs}\right)_{,j}\\ &\quad + \varepsilon_{irs}(A_{rspq}e_{pq} + B_{rspq}\kappa_{pq}).\end{aligned} \tag{7}$$

In the case that we substitute the constitutive equations (5) into the evolution equations (1), we obtain the following system:

$$\begin{aligned}\rho\ddot{u}_i &= \left(A_{jirs}e_{rs} + B_{jirs}\kappa_{rs} + A^*_{jirs}\dot{u}_{r,s}\right)_{,j},\\ I\ddot{\varphi}_i &= \left(B_{rsji}e_{rs} + C_{jirs}\kappa_{rs}\right)_{,j} + \varepsilon_{irs}(A_{rspq}e_{pq}\\ &\quad + B_{rspq}\kappa_{pq} + A^*_{rspq}\dot{u}_{p,q}).\end{aligned} \tag{8}$$

We will study systems (7) and (8) in the domain B and we will assume the boundary conditions:

$$u_i(\boldsymbol{x},t) = \varphi_i(\boldsymbol{x},t) = 0 \quad \boldsymbol{x} \in B,\quad t > 0. \tag{9}$$

[1] In the general case, we can assume that the inertia becomes a matrix; however, we assume that the matrix $I_{ij} = I\delta_{ij}$ (δ_{ij} is the Dirac symbol) and so, we make the calculations easier.

We will also impose the initial conditions, for a.e. $\boldsymbol{x} \in B$,

$$u_i(\boldsymbol{x},0) = u_i^0(\boldsymbol{x}),\quad \dot{u}_i(\boldsymbol{x},0) = v_i^0(\boldsymbol{x}),\qquad \varphi_i(\boldsymbol{x},0) = \varphi_i^0(\boldsymbol{x}),\quad \dot{\varphi}_i(\boldsymbol{x},0) = \psi_i^0(\boldsymbol{x}). \tag{10}$$

In this work, we will assume that

(i) The mass density ρ and the inertia coefficient I are strictly positives.
(ii) There exists a positive constant C such that

$$A_{ijrs}e_{ij}e_{rs} + 2B_{ijrs}e_{ij}\kappa_{rs} + C_{ijrs}\kappa_{ij}\kappa_{rs} \geq C(e_{ij}e_{ij} + \kappa_{ij}\kappa_{ij}).$$

It is also convenient to recall that

$$e_{ij}e_{ij} = u_{(i,j)}u_{(i,j)} + \varepsilon_{ijk}\varepsilon_{ijk}(\gamma_k - \varphi_k)^2,$$

where $u_{(i,j)} = \frac{1}{2}(u_{i,j} + u_{j,i})$ and $\gamma_k = \frac{1}{2}\varepsilon_{krs}u_{s,r}$.

Therefore, using the boundary conditions (9) for the displacement field we get that there exists a positive constant C^* such that

$$\int_B e_{ij}e_{ij}\,dv \geq C^*\int_B u_{i,j}u_{i,j}\,dv,$$

after the application of Korn's inequality.

3. The first problem

In this section, we will study the problem defined by system (7) with boundary conditions (9) and the initial conditions (10). We will also assume that

(iii) there exists a positive constant $D > 0$ such that

$$\int_B C^*_{ijrs}\kappa_{rs}\kappa_{ij}\,dv \geq D\int_B \kappa_{ij}\kappa_{ij}\,dv. \tag{11}$$

3.1. Existence and uniqueness

We will study the problem in the Hilbert space

$$\mathcal{H} = \boldsymbol{H}_0^1(B)\times\boldsymbol{L}^2(B)\times\boldsymbol{H}_0^1(B)\times\boldsymbol{L}^2(B),$$

where $H_0^1(B)$ and $L^2(B)$ are the usual Sobolev spaces, $\boldsymbol{H}_0^1(B) = [H_0^1(B)]^3$ and $\boldsymbol{L}^2(B) = [L^2(B)]^3$.

In this space, we will can consider the inner product

$$\begin{aligned}\langle(\boldsymbol{u},\boldsymbol{v},\boldsymbol{\varphi},\boldsymbol{\psi}),(\boldsymbol{u}^*,\boldsymbol{v}^*,\boldsymbol{\varphi}^*,\boldsymbol{\psi}^*)\rangle &= \frac{1}{2}\int_B\Big(\rho v_i\overline{v_i^*} + I\varphi_i\overline{\varphi_i^*}\\ &\quad + A_{ijrs}e_{ij}\overline{e_{rs}^*} + C_{ijrs}\kappa_{ij}\overline{\kappa_{rs}^*} + B_{ijrs}(e_{ij}\overline{\kappa_{rs}^*} + \overline{e_{ij}^*}\kappa_{rs})\Big)\,dv.\end{aligned} \tag{12}$$

As usual, we used notations (3), and the bar over a variable represents the complex conjugate.

By virtue of the assumptions previously proposed, we can guarantee that the norm associated to the inner product (12) is equivalent to the usual norm in the space $\mathcal{H}$.

Therefore, we can write our problem as a Cauchy problem:

$$\frac{d}{dt}U(t) = \mathcal{A}U(t),\quad U(0) = U^0, \tag{13}$$

where $U = (\boldsymbol{u},\boldsymbol{v},\boldsymbol{\varphi},\boldsymbol{\psi})$, $U^0 = (\boldsymbol{u}^0,\boldsymbol{v}^0,\boldsymbol{\varphi}^0,\boldsymbol{\psi}^0)$, and

$$\mathcal{A}\begin{pmatrix}\boldsymbol{u}\\ \boldsymbol{v}\\ \boldsymbol{\varphi}\\ \boldsymbol{\psi}\end{pmatrix} = \begin{pmatrix} v_i\\ \rho^{-1}[A_{ijrs}(u_{s,r} + \varepsilon_{srk}\varphi_k) + B_{ijrs}\varphi_{r,s}]_{,j}\\ \psi_i\\ I^{-1}[(B_{rsji}u_{s,r})_{,j} + \varepsilon_{irs}A_{rspq}u_{q,p}\\ +(B_{rsji}\varepsilon_{srk}\varphi_k + C_{ijrs}\varphi_{s,r} + C^*_{ijrs}\psi_{s,r})_{,j}]\end{pmatrix}.$$

We note that the domain of the operator $\mathcal{A}$ is determined by the elements of space $\mathcal{H}$ such that

$$\boldsymbol{v},\boldsymbol{\psi} \in \boldsymbol{H}_0^1(B),\quad (A_{ijrs}u_{s,r} + B_{ijrs}\varphi_{s,r})_{,j} \in \boldsymbol{L}^2(B),\qquad (B_{rsji}u_{s,r})_{,j} + (C_{ijrs}\varphi_{s,r} + C^*_{ijrs}\psi_{s,r})_{,j} \in \boldsymbol{L}^2(B).$$

It is clear that it is a dense subspace.

We also have

$$Re\langle \mathcal{A}U, U\rangle = -\frac{1}{2}\int_B C^*_{ijrs}\psi_{j,i}\psi_{s,r}\,dv \le 0, \tag{14}$$

where the inequality comes from (11). To prove the existence of a semigroup, it is sufficient to show that, for every $(\boldsymbol{f}_1, \boldsymbol{f}_2, \boldsymbol{f}_3, \boldsymbol{f}_4) \in \mathcal{H}$, there exists $(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{\varphi}, \boldsymbol{\psi})$ at the domain of the operator such that

$$\boldsymbol{v} = \boldsymbol{f}_1, \quad \boldsymbol{\psi} = \boldsymbol{f}_3,$$
$$\left(A_{ijrs}(u_{s,r} + \epsilon_{srk}\varphi_k) + B_{ijrs}\varphi_{s,r}\right)_{,j} = \rho f_{2i},$$
$$\left(B_{rsji}u_{s,r}\right)_{,j} + \epsilon_{irs}A_{rspq}u_{q,p} + \Big(B_{rsji}\epsilon_{srk}\varphi_k + C_{ijrs}\varphi_{s,r} + C^*_{ijrs}\psi_{s,r}\Big)_{,j} = I f_{4i}.$$

We can write

$$\left(A_{ijrs}(u_{s,r} + \epsilon_{srk}\varphi_k) + B_{ijrs}\varphi_{s,r}\right)_{,j} = \rho f_{2i},$$
$$\left(B_{rsij}u_{s,r}\right)_{,j} + \epsilon_{irs}A_{rspq}u_{q,p} + \left(B_{rsji}\epsilon_{srk}\varphi_k + C_{ijrs}\varphi_{s,r}\right)_{,j} = I f_{4i} - \left(C^*_{ijrs}f_{3s,r}\right)_{,j}.$$

We note that

$$\left(\rho f_{2i}, I f_{4i} - (C^*_{ijrs}f_{3s,r})_{,j}\right) \in \boldsymbol{H}^{-1}(B)\times \boldsymbol{H}^{-1}(B).$$

In view of the assumptions defined in this section and in the previous one, and the use of the Lax–Milgram lemma, we can conclude the existence of solution in the domain [23]. Therefore, zero belongs to the resolvent of the operator.

We have seen that the domain of the operator is dense, condition (14) holds and zero belongs to the resolvent. Then, we have that the operator $\mathcal{A}$ generates a contractive semigroup and we obtain the existence of solutions to the Cauchy problem (13). This is summarized in the following theorem.

Theorem 1. *For each $U^0 \in \mathcal{D}(\mathcal{A})$, there exists a unique solution*

$$U \in C^1([0,\infty);\mathcal{H}) \cap C^0([0,\infty);\mathcal{D}(\mathcal{A}))$$

to the problem (13).

It is worth noting that the existence of a semigroup provides a continuous dependence result with respect to the initial data. We could also obtain continuous dependence with respect to the supply terms in the case that we consider them.

3.2. Exponential decay

In this subsection, we will show the exponential decay of the solutions to problem (13) when we impose the following additional assumption:

(iv) There exists a positive constant D^* such that

$$\int_B \left(B_{rsji}u_{s,r}u_{i,j} - \epsilon_{irs}A_{rspq}u_{q,p}u_i\right)dv \ge D^*\int_B u_{i,j}u_{i,j}\,dv, \tag{15}$$

for every vector field u_i vanishing at the boundary.[2]

At the same time, the analysis also holds if we assume that

$$\int_B \left(B_{rsji}u_{s,r}u_{i,j} - \epsilon_{irs}A_{rspq}u_{q,p}u_i\right)dv \le -D^*\int_B u_{i,j}u_{i,j}\,dv.$$

Remark 1. In order to impose condition (iv) we can assume that there exist two positive constants M_1 and M_2 such that

$$\int_B B_{rsji}u_{s,r}u_{i,j}\,dv \ge M_1\int_B u_{i,j}u_{i,j}\,dv, \tag{16}$$

$$\left|\int_B \epsilon_{irs}A_{rspq}u_{q,p}u_i\,dv\right| \le M_2\left(\int_B u_{i,j}u_{i,j}\,dv\right)^{1/2}\left(\int_B u_iu_i\,dv\right)^{1/2}. \tag{17}$$

After the use of the Poincaré inequality, we could propose sufficient conditions on M_1 and M_2 and the geometry of the domain to guarantee condition (iv).

In the case that

$$\epsilon_{irs}A_{rspq} = \epsilon_{qrs}A_{rspi},$$

we can see that

$$\int_B \epsilon_{irs}A_{rspq}u_{q,p}u_i\,dv = \frac{-1}{2}\int_B (\epsilon_{irs}A_{rspq})_{,p}u_qu_i\,dv.$$

Therefore, whenever $\left(\epsilon_{irs}A_{rspq}\right)_{,p}u_qu_i$ is semi-definite positive, it is sufficient to impose condition (16). We note that, in the homogeneous case, this condition is satisfied.

Now, we are going to prove the result of this subsection.

Theorem 2. *The semigroup generated by the operator $\mathcal{A}$ defined previously is exponentially stable; that is, there exist two positive constants M and ω such that*

$$\|U(t)\|_{\mathcal{H}} \le Me^{-\omega t}\|U(0)\|_{\mathcal{H}}.$$

Proof. We will show the theorem by using the condition which ensures the exponential decay whenever the imaginary axis is contained at the resolvent of the operator and the condition

$$\overline{\lim}_{|\lambda|\to\infty}\|(i\lambda\mathcal{I} - \mathcal{A})^{-1}\| < \infty$$

holds.

In order to obtain the first condition, we proceed by contradiction. Therefore, taking into account that zero belongs to the resolvent of the operator, if we assume that this condition does not hold, there will exist a sequence of real numbers $\lambda_n \to \tau \neq 0$ and a sequence of unit norm vectors U_n, in the domain of the operator $\mathcal{A}$, such that

$$\|(i\lambda_n\mathcal{I} - \mathcal{A})U_n\| \to 0.$$

This condition is equivalent to the following convergences:

$$i\lambda u - v \to 0 \quad \text{in} \quad \boldsymbol{H}^1_0(B),$$
$$i\rho\lambda v - \left(A_{ijrs}(u_{s,r} + \epsilon_{srk}\varphi_k) + B_{ijrs}\varphi_{s,r}\right)_{,j} \to 0 \quad \text{in} \quad \boldsymbol{L}^2(B),$$
$$i\lambda\varphi - \boldsymbol{\psi} \to 0 \quad \text{in} \quad \boldsymbol{H}^1_0(B),$$
$$iI\lambda\psi - \left[(B_{rsji}u_{s,r})_{,j} + \epsilon_{irs}A_{rspq}u_{q,p} + \left(B_{rsji}\epsilon_{srk}\varphi_k + C_{ijrs}\varphi_{s,r} + C^*_{ijrs}\psi_{s,r}\right)_{,j}\right] \to 0 \quad \text{in} \quad \boldsymbol{L}^2(B).$$

Here, we have omitted the sub-index "n" for the sake of simplicity in the notation. If we take into account the dissipation inequality (11) we find that $\boldsymbol{\psi} \to \boldsymbol{0}$ in $\boldsymbol{H}^1_0(B)$, and so, we also have $\boldsymbol{\varphi} \to 0$ in $\boldsymbol{H}^1_0(B)$. If we multiply the last convergence by $\boldsymbol{u}$ and we take into account that

$$\langle iI\lambda\boldsymbol{\psi}, \boldsymbol{u}\rangle = \langle iI\boldsymbol{\psi}, \lambda u\rangle \to 0,$$

we obtain that

$$\int_B \left(B_{rsji}u_{s,r}u_{i,j} - \epsilon_{irs}A_{rspq}u_{q,p}u_i\right)dv \to 0.$$

Keeping in mind condition (iv) we find that $\boldsymbol{u} \to \boldsymbol{0}$ in $\boldsymbol{H}^1_0(B)$. If we multiply now the second convergence by $\boldsymbol{u}$ we obtain that $\boldsymbol{v} \to \boldsymbol{0}$ in $\boldsymbol{L}^2(B)$. Therefore, we arrive to a contradiction because we assumed that there was a point of the spectrum at the imaginary axis.

In order to show the asymptotic condition, we can use a similar argument assuming again that it does not hold. In fact, in this case there will exist a sequence of unit norm vectors U_n at the domain of the operator and a sequence of real numbers $\lambda_n \to \infty$ such that the above convergences hold. The arguments used to prove that the imaginary axis was contained at the resolvent can also be employed here in a similar form. The unique point that we must keep in mind is that λ_n does not tend to zero.

[2] It is relevant to point out that we need to assume that B_{ijrs} has a "strict" definite sign. For isotropic and centrosymmetric solids, this pseudotensor does not appear in the constitutive equations. Therefore, our result can be applied only for anisotropic materials.

4. The second problem

In this section, we will sketch in a quick way how we could obtain the exponential decay of the solutions to the problem determined by system (8) with boundary conditions (9) and initial conditions (10).

First, we must assume the conditions imposed in Section 2, but now we change condition (11) to assume

(iii*) there exists a positive constant D^* such that

$$\int_B A^*_{ijrs} u_{j,i} u_{s,r} \, dv \leq -D^* \int_B u_{i,j} u_{i,j} \, dv.$$

Under the conditions proposed in Section 2 and the above condition, we could obtain straightforwardly a similar result to Theorem 1.

We could also derive an exponential energy decay (see Theorem 2) if we assume that

(iv*) there exists a positive constant D^{**} such that

$$\int_B \left(B_{ijrs} \varphi_{s,r} \varphi_{i,j} + \epsilon_{srk} A_{ijrs} \varphi_k \varphi_{i,j} \right) dv \geq D^{**} \int_B \varphi_{i,j} \varphi_{i,j} \, dv,$$

in place of condition (15).

Analogously, we can also impose that

$$\int_B \left(B_{ijrs} \varphi_{s,r} \varphi_{i,j} + \epsilon_{srk} A_{ijrs} \varphi_k \varphi_{i,j} \right) dv \leq -D^{**} \int_B \varphi_{i,j} \varphi_{i,j} \, dv.$$

An argument similar to Remark 1 can be done in this case.

In order to avoid the repetition of the arguments used in the previous section, we will skip the details. Therefore, we can conclude the following.

Theorem 3. *For each $U^0 \in \mathcal{D}(\mathcal{A})$, there exists a unique solution*

$$U \in C^1([0,\infty); \mathcal{H}) \cap C^0([0,\infty); \mathcal{D}(\mathcal{A}))$$

to the corresponding modification of problem (13). Moreover, this solution is exponentially stable; that is, there exist two positive constants M and ω such that

$$\|U(t)\|_{\mathcal{H}} \leq M e^{-\omega t} \|U(0)\|_{\mathcal{H}}.$$

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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