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Fractional model of COVID-19 applied to Galiza, Spain and Portugal

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Abstract

A fractional compartmental mathematical model for the spread of the COVID-19 disease is proposed. Special focus has been done on the transmissibility of super-spreaders individuals. Numerical simulations are shown for data of Galiza, Spain, and Portugal. For each region, the order of the Caputo derivative takes a different value, that is not close to one, showing the relevance of considering fractional models.

Keywords: mathematical modeling of COVID-19 pandemic, Galiza, Spain and Portugal case studies, fractional differential equations, numerical simulations.

2010 MSC: 26A33, 34A08, 92D30.

1. Introduction

Coronavirus disease 2019 (COVID-19), the outbreak due to severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), has now taken on pandemic proportions in 2020, affecting several millions of individuals in almost all countries [12]. An Integrated Science and multidisciplinary approach is necessary to fight the COVID-19 pandemic [17, 18]. In particular, mathematical and epidemiological simulation plays a crucial role in predicting, anticipating, and controlling present and future epidemics.

As for the mathematical modelling of coronavirus disease COVID-19, it has been shown to be extremely useful for governments in order to define appropriate policies [19]. In this direction, a number of papers has been published recently related with modelling of this pandemic (see, e.g., [6, 9], just to cite some of them). In [19] a model including the super-spreader class has been presented, and applied to give an estimation of the infected and death individuals in Wuhan. The collaboration with Galician government [3] has allowed us to understand some important considerations in order to perform analysis. In particular, due to the pandemic, some cases have not been reported as expected, but with some days of delay. As a consequence, in this paper we propose to consider not the daily reported cases, but the means in the previous 5 days of daily reported cases. As a consequence, it seems appropriate to consider fractional derivatives, which have been intensively used to obtain models of infectious diseases since they take into account the memory effect, which is now bigger due to the aforementioned mean of the five previous days of daily reported cases. Having estimates *a priori* of infected individuals of COVID-19, obtained by using mathematical models, have helped to predict the number of required beds both for hospitalized individuals and mainly at intensive care units [3].

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Fractional calculus and fractional differential equations have recently been applied in numerous areas of mathematics, physics, engineering, bio-engineering, and other applied sciences. We refer the reader to the monographs [11, 13, 22, 24, 27, 25, 7] and the articles [1, 2, 26, 20]. In this work we shall consider the Caputo fractional derivative [4] (see also [8]). A fractional model using Caputo-Fabrizio fractional derivative of COVID-19 in Wuhan (China) has been developed in [21].

The structure of this work is as follows. In Section 2, we introduce a fractional model by using Caputo fractional derivatives of the classical compartmental model presented in [19]. In Section 3, some numerical results are presented for three different territories: Galiza, Spain, and Portugal. Galiza is an autonomous community of Spain and located in the northwest Iberian peninsula and having a population of about 2,700,000 and a total area of 29,574 km2. Spain (officially, Kingdom of Spain) is a country mostly located on the Iberian Peninsula, in southwestern Europe, with a population of about 47,000,000 people and a total area of 505,992 km2. Portugal (officially, Portuguese republic) is also a country located mostly on the Iberian Peninsula with a population of about 10,276,000 individuals and a total area of 92,212 km2. We end with Section 4 of conclusions and discussion.

2. The Proposed COVID-19 fractional model

In what follows we shall assume that we have a constant population divided in 8 epidemiological classes, namely:

1. susceptible individuals (S),
2. exposed individuals (E),
3. symptomatic and infectious individuals (I),
4. super-spreaders individuals (P),
5. infectious but asymptomatic individuals (A),
6. hospitalized individuals (H),
7. recovery individuals (R), and
8. dead individuals (F) or fatality class.

Our model is based on the one presented in [19] and substituting the first order derivative by a derivative of fractional order. We use the fractional derivative in the sense of Caputo: for an absolutely continuous function $f : [0, \infty) \to \mathbf{R}$ the Caputo fractional derivative of order $\alpha > 0$ is given by [11, 13, 16, 22]:

$$D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-s)^{-\alpha} f'(s) ds.$$

Fractional calculus and fractional differential equations are an active area of research and in some cases adequate to incorporate the history of the processes [1, 10, 14, 15, 16, 23, 27]. The fractional proposed model takes the form

$$\begin{cases} D^\alpha S(t) = -\beta \dfrac{I}{N} S - l\beta \dfrac{H}{N} S - \beta' \dfrac{P}{N} S, \\ D^\alpha E(t) = \beta \dfrac{I}{N} S + l\beta \dfrac{H}{N} S + \beta' \dfrac{P}{N} S - \kappa E, \\ D^\alpha I(t) = \kappa \rho_1 E - (\gamma_a + \gamma_i) I - \delta_i I, \\ D^\alpha P(t) = \kappa \rho_2 E - (\gamma_a + \gamma_i) P - \delta_p P, \\ D^\alpha A(t) = \kappa (1 - \rho_1 - \rho_2) E, \\ D^\alpha H(t) = \gamma_a (I + P) - \gamma_r H - \delta_h H, \\ D^\alpha R(t) = \gamma_i (I + P) + \gamma_r H, \\ D^\alpha F(t) = \delta_i I(t) + \delta_p P(t) + \delta_h H(t), \end{cases} \tag{1}$$

in which we have the following parameters:

1. β quantifies the human-to-human transmission coefficient per unit time (days) per person,
2. β' quantifies a high transmission coefficient due to super-spreaders,
3. l quantifies the relative transmissibility of hospitalized patients,
4. κ is the rate at which an individual leaves the exposed class by becoming infectious (symptomatic, super-spreaders or asymptomatic),
5. ρ_1 is the proportion of progression from exposed class E to symptomatic infectious class I,
6. ρ_2 is a relative very low rate at which exposed individuals become super-spreaders,
7. $1 - \rho_1 - \rho_2$ is the progression from exposed to asymptomatic class,
8. γ_a is the average rate at which symptomatic and super-spreaders individuals become hospitalized,
9. γ_i is the recovery rate without being hospitalized,
10. γ_r is the recovery rate of hospitalized patients,
11. δ_i denotes the disease induced death rates due to infected individuals,
12. δ_p denotes the disease induced death rates due to super-spreaders individuals,
13. δ_h denotes the disease induced death rates due to hospitalized individuals.

A flowchart of the classical model (1) is presented in Figure 2. For additional details and particular values of the parameters we refer the reader to [19].

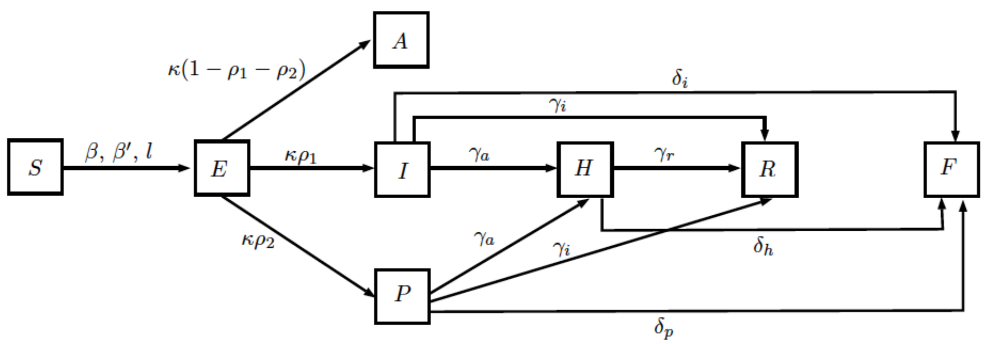

Figure 1: Flowchart of model (1).

3. Numerical Simulations

Next we shall show the numerical simulations in three territories: Galiza, Portugal, and Spain. For all these cases we have considered the official data published by the corresponding authorities and we have computed the means of the five previous reports. As it has been observed during this pandemic, the output of the laboratories has had some delays due to the big pressure and collapse of the public systems. In this way, some cases have been reported with some delay and some updates have been published days later of the corresponding dates. In order to reduce these problems, we consider the mean of the five previous reported cases, always following the official data. Moreover, in each of the territories there are specificities such as territorial dispersion/concentration, use of public transportation, and mainly the date of starting the confinement as compared with the initial spread of the COVID-19. These factors imply tiny adjustments in the factor to divide the total population as well as in the value of the fractional parameter α, respectively. For solving the system of fractional differential equations (1) we have used [5], by using Matlab in a MacBook Pro computer with a 2.3 GHz Intel Core i9 processor and 16 GB of memory 2400 MHz DDR4.

Date	Confirmed	New confirmed	5 days mean		Date	Confirmed	New confirmed	5 days mean
03-08	6	1	1		04-03	5625	406	380,4
03-09	22	16	4,2		04-04	5944	319	381
03-10	35	13	6,4		04-05	6151	207	343,8
03-11	35	0	6,4		04-06	6331	180	297,8
03-12	85	50	16		04-07	6538	207	263,8
03-13	115	30	21,8		04-08	6758	220	226,6
03-14	195	80	34,6		04-09	6946	188	200,4
03-15	245	50	42		04-10	7176	230	205
03-16	292	47	51,4		04-11	7336	160	201
03-17	341	49	51,2		04-12	7494	158	191,2
03-18	453	112	67,6		04-13	7597	103	167,8
03-19	578	125	76,6		04-14	7708	111	152,4
03-20	739	161	98,8		04-15	7873	165	139,4
03-21	915	176	124,6		04-16	8013	140	135,4
03-22	1208	293	173,4		04-17	8084	71	118
03-23	1415	207	192,4		04-18	8185	101	117,6
03-24	1653	238	215		04-19	8299	114	118,2
03-25	1915	262	235,2		04-20	8468	169	119
03-26	2322	407	281,4		04-21	8634	166	124,2
03-27	2772	450	312,8		04-22	8805	171	144,2
03-28	3139	367	344,8		04-23	8932	127	149,4
03-29	3723	584	414		04-24	9116	184	163,4
03-30	4039	316	424,8		04-25	9176	60	141,6
03-31	4432	393	422		04-26	9238	62	120,8
04-01	4842	410	414		04-27	9328	90	104,6
04-02	5219	377	416					

Table 1: Data of the autonomous region of Galiza. The list of 51 days includes the cumulative, new infected and mean of the previous 5 days.

3.1. The Case Study of Galiza

In the autonomous region of Galiza we have the values given in Table 3.1 as for the cumulative cases, the new daily infected individuals, as well as the mean of the 5 previous days. The list includes 51 values starting 7th March since after that date (27th April) the way of officially computing individuals has changed.

By considering the fractional order $\alpha = 0.85$ and the same values of the parameters as in [19], the results of the numerical simulation are shown in Figure 3.1. The green line denotes the real data while the black line is the numerical solution of the fractional system (1), with total population $N = 2,700,000/500$, where $N = S + E + I + P + A + H + R + F$, since the population of Galiza is widely dispersed in the territory with very few big cities and low use of public transportation.

3.2. The Case Study of Spain

As for the Kingdom of Spain, the data of 82 days is collected in Table 3.2, as for the cumulative cases, the new daily infected individuals, as well as the mean of the 5 previous days, starting 25th February.

By considering again the fractional order $\alpha = 0.85$ and the same values of the parameters as in [19], the results of the numerical simulation are shown in Figure 3.2. The green line denotes the real data while the black line is the numerical solution of the fractional system (1), with $N = 47,000,000/425$ since in some parts of Spain there is more concentrated population and intensive use of public transportation.

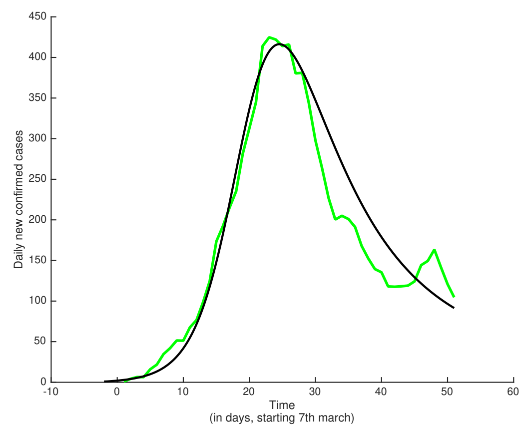

Figure 2: Number of confirmed cases per day in Galiza. The green line corresponds to the real data given in Table 3.1 while the black line $(I + P + H)$ has been obtained by solving numerically the system of fractional differential equations (1), by using [5].

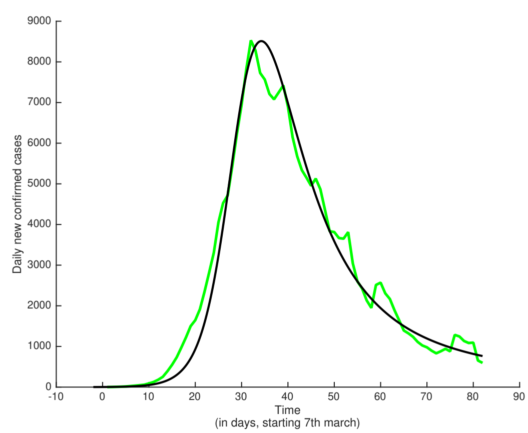

Figure 3: Number of confirmed cases per day in Spain. The green line corresponds to the real data given Table 3.2 while the black line $(I + P + H)$ has been obtained by solving numerically the system of fractional differential equations (1), by using [5].

3.3. The Case Study of Portugal

As for the Republic of Portugal, the data of 56 days starting 3rd March for the cumulative cases, the new daily infected individuals, as well as the mean of the 5 previous days is collected in Table 3.3.

By considering now the fractional order $\alpha = 0.75$ and the same values of the parameters as in [19], the results of the numerical simulation are shown in Figure 3.3. As in the previous figures, the green line denotes the real data while the black line is the numerical solution of the fractional system (1), with $N = 10,280,000/1750$ since the Portuguese population is widely dispersed and the confinement started at an earlier stage of the spread of the disease.

4. Conclusions and Discussion

In this paper, we have shown the importance of considering a fractional Caputo differential system, where the order of the derivative α plays a crucial role to fit the number of confirmed cases in the regions of Galiza, Spain and Portugal. In fact, the considered values of $\alpha = 0.85$ for Galiza and Spain and $\alpha = 0.75$ for Portugal, are not close to 1 (the classical derivative), as it

Date	Confirmed	New confirmed	5 days mean		Date	Confirmed	New confirmed	5 days mean
02-25	10	6	1,4		04-06	147717	5213	5676,2
02-26	18	8	3		04-07	153303	5586	5337,4
02-27	36	18	6,6		04-08	159051	5748	5151,4
02-28	55	19	10,4		04-09	163591	4540	4951,8
02-29	83	28	15,8		04-10	168151	4560	5129,4
03-01	138	55	25,6		04-11	172054	3903	4867,4
03-02	195	57	35,4		04-12	175087	3033	4356,8
03-03	270	75	46,8		04-13	178224	3137	3834,6
03-04	352	82	59,4		04-14	182662	4438	3814,2
03-05	535	183	90,4		04-15	186484	3822	3666,6
03-06	769	234	126,2		04-16	190308	3824	3650,8
03-07	1101	332	181,2		04-17	194150	3842	3812,6
03-08	1536	435	253,2		04-18	193437	713	3042,6
03-09	2309	773	391,4		04-19	195655	2218	2598,6
03-10	3285	976	550		04-20	198614	2959	2426
03-11	4442	1157	734,6		04-21	200968	2354	2132
03-12	5976	1534	975		04-22	203888	2920	1947,6
03-13	7659	1683	1224,6		04-23	206002	2114	2513
03-14	9806	2147	1499,4		04-24	208507	2505	2570,4
03-15	11515	1709	1646		04-25	210148	1641	2306,8
03-16	14018	2503	1915,2		04-26	211807	1659	2167,8
03-17	17713	3695	2347,4		04-27	213338	1531	1890
03-18	21764	4051	2821		04-28	214215	877	1642,6
03-19	26333	4569	3305,4		04-29	215470	1255	1392,6
03-20	31779	5446	4052,8		04-30	216757	1287	1321,8
03-21	36645	4866	4525,4		05-01	217992	1235	1237
03-22	41291	4646	4715,6		05-02	218894	902	1111,2
03-23	48984	7693	5444		05-03	219338	444	1024,6
03-24	57546	8562	6242,6		05-04	220362	1024	978,4
03-25	66503	8957	6944,8		05-05	221236	874	895,8
03-26	75691	9188	7809,2		05-06	222145	909	830,6
03-27	83944	8253	8530,6		05-07	223305	1160	882,2
03-28	90371	6427	8277,4		05-08	224048	743	942
03-29	96184	5813	7727,6		05-09	224755	707	878,6
03-30	104332	8148	7565,8		05-10	227659	2904	1284,6
03-31	111745	7413	7210,8		05-11	228373	714	1245,6
04-01	119336	7591	7078,4		05-12	228978	605	1134,6
04-02	126616	7280	7249		05-13	229471	493	1084,6
04-03	133294	6678	7422		05-14	230228	757	1094,6
04-04	138832	5538	6900		05-15	230929	701	654
04-05	142504	3672	6151,8		05-16	231350	421	595,4

Table 2: Data of the Kingdom of Spain. The list of 82 days includes the cumulative, new infected and mean of the previous 5 days.

happens in many of the proposed fractional compartmental models in the literature. Note that the same values of the parameters in the differential system (1), taken from [19], were used for the three regions. Therefore, we may conclude that model (1) can be used to approximate the confirmed cases of COVID-19 in regions with different economic, geographical, social and epidemic characteristics, as it happens for the three considered regions in this paper.

Date	Confirmed	New confirmed	5 days mean		Date	Confirmed	New confirmed	5 days mean
03-03	4	2	4		03-31	7443	1035	725,9
03-04	6	2	2		04-01	8251	808	750,9
03-05	9	3	3		04-02	9034	783	784,3
03-06	13	4	4		04-03	9886	852	802,6
03-07	21	8	8		04-04	10524	638	764,9
03-08	30	9	9		04-05	11278	754	759,4
03-09	39	9	5,5		04-06	11730	452	760,3
03-10	41	2	5,3		04-07	12442	712	714,1
03-11	59	18	7,6		04-08	13141	699	698,6
03-12	78	19	9,9		04-09	13956	815	703,1
03-13	112	34	14,1		04-10	15472	1516	798
03-14	169	57	21,1		04-11	15987	515	780,4
03-15	245	76	30,7		04-12	16585	598	758,1
03-16	331	86	41,7		04-13	16934	349	743,4
03-17	448	117	58,1		04-14	17448	514	715,1
03-18	642	194	83,3		04-15	18091	643	707,1
03-19	785	143	101		04-16	18841	750	697,9
03-20	1020	235	129,7		04-17	19022	181	507,1
03-21	1280	260	158,7		04-18	20206	1184	602,7
03-22	1600	320	193,6		04-19	20863	657	611,1
03-23	2060	460	247		04-20	21379	516	635
03-24	2362	302	273,4		04-21	21982	603	647,7
03-25	2995	633	336,1		04-22	22353	371	608,9
03-26	3544	549	394,1		04-23	22797	444	565,1
03-27	4268	724	464		04-24	23392	595	624,3
03-28	5170	902	555,7		04-25	23864	472	522,6
03-29	5962	792	623,1		04-26	24027	163	452
03-30	6408	446	621,1		04-27	24322	295	420,4

Table 3: Data of the Republic of Portugal. The list of 56 days includes the cumulative, new infected and mean of the previous 5 days.

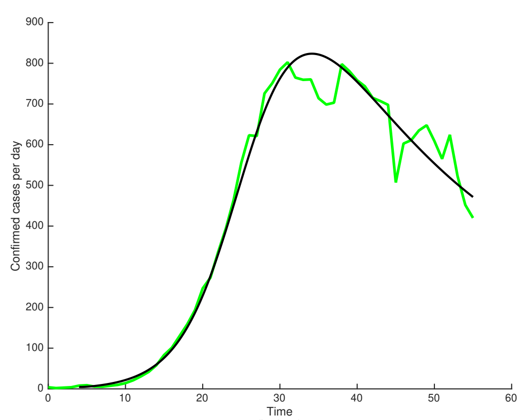

Figure 4: Number of confirmed cases per day in Portugal. The green line corresponds to the real data given in Table 3.3 while the black line $(I + P + H)$ has been obtained by solving numerically the system of fractional differential equations (1), by using [5].

Our variables are divided into eight epidemiological sub-populations as in [19] and they are detailed at the beginning of the second section dedicated to the introduction of the dynamical model. We have solved our fractional dynamical model using a subroutine called FracPECE [5] to approximate numerically the solution of the fractional system of differential equations proposed. Our numerical simulations show a good agreement between the output of the fractional model given by the sum of the symptomatic and infectious individuals, super- spreaders and hospitalized individuals and the data collected form the health authorities in Spain, Portugal and Galicia. We plan to consider other countries and regions in our future studies and also, of course, an update of the data.

In the future we will study the stability of the possible equilibrium point, the bifurcation of solutions depending on the parameters and the role of the basic reproduction number.

The fractional model is novel and in the future we will study the optimal fractional order of derivation for the study of the COVID-19 epidemic in different contexts. The system has a unique solution for given initial conditions and a detailed mathematical analysis study will be performed. A crucial point is, of course, to determine the optimal fractional order α adequate for each process and, in this case, each region.

We will continue our research using this and other future models as well as considering different approaches as the COVID-19 evolves.

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