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Applying the Shapley value to the tuna fishery

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A B S T R A C T

The tuna fishing sector has faced important regulatory restrictions for years, mainly based on the number of fish aggregating devices (FADs) allowed per vessel, which has threatened the survival of many tuna firms. Various academics have studied this issue, proposing various solutions based on the reassignment and sharing of FADs. However, previous research has focused primarily on the use of FADs and their implications, rather than actually helping to optimize the tuna fleet's fishing activity, and possibly for this reason, none of these proposals has impacted current fishing practices. In light of this situation, our research proposes a more equitable approach: we have modeled the tuna vessel problem as a cooperative game, reallocating FADs among vessels, studying the Shapley value, and comparing the results achieved with previous proposals. Although our approach is fairly standard in the literature, it is a novel solution to a deep-rooted problem in this sector that also leads to a significant reduction in CO_2 emissions associated with fuel consumption. In fact, the application of our theoretical results to real data shows that there is not only a significant scope for improvement for firms and their vessels –both gain more revenue– but also a beneficial contribution to the environment in terms of reduced fuel consumption.

1. Introduction

Our research explores how to improve tuna firms profits through cooperation among their vessels while reducing CO_2 emissions into the atmosphere. With this idea in mind, we model the tuna fishing problem as a cooperative game by studying the Shapley value. Our goal is to propose a fairer way of sharing the total revenue between vessels and their firms, which would also help to reduce pollution.

We focus our study on the case of tropical tuna vessels, as this type of vessel mostly uses fish aggregating devices (FADs) in their daily work. These tuna vessels usually work in small groups (two or three) belonging to the same firm (Groba, Sartal, & Bergantiños, 2020). The *modus operandi* they follow is simple. First, each ship releases (or "harvests") its FADs in the ocean, in specific positions chosen by the skipper based on his experience. Once released, the FADs drift in the ocean, but the location chosen by the skipper will be key to the fishery's success. In fact, once the FADs are "planted", each vessel retrieves only its own FADs, and an important part of the salary is a function of the catches made. It should be noted that the firm that owns the tuna vessels is responsible for all running costs, among which fuel consumption is by far the most important.

Thus, taking into account the working operations, it seems evident that any firm would be interested in the possibility of sharing and reallocating FADs among the vessels that make up its fleet. This would reduce the total costs by minimizing the distance traveled (and the associated fuel), but maintain profits by distributing a part to the skippers and crew members of the tuna vessels. In fact, there are several authors who, aware of this situation, have proposed various mechanisms for encouraging the reassignment and sharing of FADs. For example, Groba et al. (2020) proposed noncooperative mechanisms of FAD reallocation based on Bayesian Nash equilibria. The firm offers a guarantee to the vessels that share its FADs: the firm will compensate those vessels that are harmed by the reassignment, so that they reach at least the amount initially allocated with their FADs.

Given initiatives such as the one proposed, one would think that the reallocation of FADs between vessels would be a common practice in the sector. However, the tuna grouping highlights that the adoption of this initiative type has been low or even nonexistent to date. The fact that profit maximization is not always ensured, combined with skippers' refusal to share their know-how as FAD harvesters without commensurate rewards, are the main reasons for this initiative type's failure.

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Aware of this situation, we propose in our research a different approach to the problem of allocating revenues from FAD sharing: cooperative game theory. First, we associate each tuna fishing problem with a cooperative game and compute the Shapley value corresponding to that cooperative game. Then, we allocate revenue using the calculated Shapley value. Although our approach is fairly standard in the literature, it is a novel solution to a problem that is deeply rooted in this sector, and it allows for a considerable reduction in the CO_2 emissions associated with fuel consumption. We find inspiration for this problem's solution in classic cases, such as airport problems, where the cost of a runway has to be shared among aircraft (Littlechild & Owen, 1973), or bankruptcy problems (O'Neill, 1982). Recent examples include revenue sharing for broadcasting sports leagues (Bergantiños & Moreno-Ternero, 2020; 2022), the minimum cost spanning tree problem (Bergantiños & Vidal-Puga, 2021), or how to allocate venture capital (Boonen, De Waegenaere, & Norde, 2020). The book by Algaba, Fragnelli, & Sánchez-Soriano (2019) has a collection of papers exploring several applications of Shapley's value.

The operation of the cooperative game associated with each tuna vessel problem is as follows. In the case of non-sharing, each vessel recovers its FADs, and the firm pays them a percentage based on the amount of tuna caught. As previously explained, the additional revenue from sharing comes from the reduction of fuel consumption by the vessels, motivated by the reallocation of FADs among the vessels. As the fuel is paid by the firm, only coalitions within the firm can earn revenue from sharing. Therefore, coalitions that are not within the firm obtain the same revenue as those in the non-sharing case. For each set of vessels S, the value of the coalition between S and the firm is calculated under the following assumptions. Each vessel outside of S retrieves its FADs and sells them to the firm. The FADs allocated to S are reallocated among the vessels in S by minimizing the total distance traveled. Each vessel in S recovers its reallocated FADs. The fuel cost of all vessels is paid by the firm.

Accordingly, we study some of the Shapley value's theoretical properties that are interesting for the various casuistries of the real problem considered in this research. Proposition 1 states that the Shapley value guarantees each agent (the vessels and the firm) at leastthe same revenues as those in the non-sharing case. That is, it provides incentives to cooperate. Proposition 2 says that the Shapley value of each vessel can be decomposed as the sum of two parts. The first depends on the amount of tuna recovered in its FAD. The second comes from fuel savings. Similarly, the firm's Shapley value depends on two parts: the amount of tuna recovered in all FADs and fuel savings. Proposition 3 states that, in the case of two vessels, the Shapley value has a particular expression. The extra revenue obtained by the cooperation is divided equally between the vessels and the firm. If we calculate the difference between the Shapley value and the revenue obtained without cooperation for each agent, we realize that this difference is the same for all agents.

In practice, to apply the allocation obtained through the Shapley value could be complicated because agents should bargain among them to achieve agreements. Nevertheless, in some cases, it is possible to implement the Shapley value through a mechanism in which no direct agreement among agents is needed. Namely, each agent takes a decision involving itself, and if all agents decide rationally, then the allocation obtained through the mechanism is the Shapley value. Thus, our paper is related with the literature on implementation of the Shapley value as, for instance, Perez-Castrillo & Wettstein (2001) studying cooperative games with transferable utility, Bergantiños & Vidal-Puga (2010) studying minimum cost spanning tree problems, Ju, Chun, & van den Brink (2014) studying the queuing problem, and Liu, Tsay, & Yeh (2022) studying the sequencing problem. We consider a simple four-stage mechanism which is modeled as a non-cooperative game in extensive form. We prove that the Shapley value of the cooperative game can be obtained as the payoff allocation given by a subgame perfect Nash equilibrium of the non-cooperative game.

The non-cooperative game has four stages. At Stage 1, each vessel independently decides if it shares its FADs with other vessels. At Stage 2, the firm reassigns the FADs among the vessels that decided to share it. The other vessels maintain the initial assignment. At Stage 3, the vessels fish in the new assigned FADs. At Stage 4, the firm pays the vessels. Vessels that decided not to share their FADs are paid according to the amount recovered. Vessels that decided to share their FADs are paid according to the Shapley value.

Finally, we apply the proposed theoretical development to real examples. Our data come from different groups of tuna vessels retrieving their FADs in the Indian Ocean during April 2018. In this context, we compute the Shapley value to show that a fair assignment can be found with respect to FAD sharing among vessels, and we compare the results with those of previous research to demonstrate their validity. When the right incentives are found, there is room for overall improvement for all parties, including the skippers (crew), firm, and environment. In this study, we demonstrate that the trade-off between the optimization and equity faced by tuna fishing firms is achievable. Because all of a firm's FADs can be shared fairly, the best outcome for the firm, the skippers, and the environment can be achieved.

The paper is organized as follows. In Section 2, we carefully explain the different elements of the tuna fishing vessel problem and previous approaches. In Section 3, we introduce the cooperative game and present the results related to the Shapley value. In Section 4, we implement the Shapley value through a non-cooperative game with four stages. In Section 5, we make an empirical analysis, and finally, we conclude Section 6 by highlighting the paper's main contributions and implications.

2. The tuna fishing vessels problem

2.1. Empirical problem and previous approaches

The tropical tuna industry is one of the largest and most important fishing industries in the world, both in volume and in revenue. Tuna fishing is practiced in all oceans of the world, and the industry has grown steadily during the past 60 years. The purse seine is the most commonly used and fastest growing fishing gear targeting tuna (Parker, Vázquez-Rowe, & Tyedmers, 2015). In the open ocean, many species, including tuna, interact with drifting objects on the surface, such as logs or branches, due to food webs that are spontaneously generated under these types of objects (Dempster & Taquet, 2004).

Knowing this, since the mid-1980s, skippers have begun to experiment with ways in which to maximize the potential of floating objects as fishing tools. At first, reflectors and radio beacons were attached to logs to improve detection at greater distances, and eventually, fishermen began to build drifting FADs equipped with electronic FADs to increase the number of floating objects in the ocean and aid in their detection (Davies, Mees, & Milner-Gulland, 2014). Since then, the technology used to track FADs has continued to evolve steadily: from radio-based FADs to GPS-enabled FADs that use satellite communication and are equipped with echo sounders to estimate the amount of biomass aggregated under the FADs (Groba et al., 2020). The advantage of using FADs to increase tuna catches, coupled with the technological evolution of FADs, caused the purse seine industry to increase the use of FADs, even for fleets that had traditionally relied on free school (Lopez, Moreno, Sancristobal, & Murua, 2014).

The immediate consequence of this technological evolution was a considerable increase in the number of catches in a short period, which led regional fisheries management organizations (RFMOs) to increase regulations for FAD fishing. Thus, a primary constraint currently affecting tuna vessels is the reduction in the number of FADs that can be handled at the same time. Without questioning the need for this type of initiative, it should be noted that these limitations are stifling for many small firms due to the reduction in income and, above all, the continuous increase in costs, especially those derived from fuel consumption.

This is the starting point of our research. As the FADs are based on satellite technology, the FAD information can be shared, which means that not only can the FAD owner receive this information but also other vessels can. When a group of vessels receive the same FAD information, it is considered to be a shared group. When this occurs, tuna vessels may share their FADs with other vessels, especially in cases where they all belong to the same firm. However, this rarely happens despite the interesting economic and environmental implications.

Authors such as Groba et al. (2020) have shown that FAD sharing helps firms to improve fishing information among vessels and also leads to certain profitability improvements due to the fuel savings realized when vessels operate in large sea areas. They propose a mechanism called "FAD reassignment with compensation" (i.e., a non-cooperative game with incomplete information based on the Bayesian Nash equilibrium). The firm offers a guarantee to vessels that share their FADs: the firm will pay, at a minimum, the original number of FADs allocated to the vessel. For example, if a vessel that initially had 20 FADs, shares them and is reassigned with 18, the firm pays to assume the remaining two FADs. If the vessel is reassigned with the same or more FADs than it had initially, it will be paid according to the number of FADs reassigned. This shows that the revenues for the firm and the individual vessels are equal or higher when FADs are shared. Moreover, the expected revenue that each vessel obtains by sharing its FADs, regardless of the FADs' positions and other vessels' decisions, is never lower than the expected utility they obtain by not sharing them.

Although this mechanism is found to be suitable for the firm and the vessels, the aggregate revenues of all agents may not be maximized in some cases. In addition, the distribution may not be entirely equitable because it does not explicitly take into account the contribution from each player (firm and tuna skippers) to the final result.

It is possibly because of these issues that, despite the proven benefits for the firm, this practice is not as widespread as one would expect, and the exchange is limited exclusively to small groups of vessels based on personal relationships and trust between skippers. Nevertheless, it should not be forgotten that the incentives of the purse seine crew are clear: they depend mainly on the amount of tuna they catch, and maximizing catches is their central objective. However, the firm's incentives go beyond the quantity of fish, including all crew costs, supplies, goods, and fuel, among others. As the cost of fuel is borne entirely by the firm, the vessels –more specifically, the skippers– have no direct incentive to share their FADs with the other vessels to minimize the distance traveled. However, the firm does have incentives. To the extent that fuel consumption is reduced due to fewer trips during FAD collection, revenues increase. In addition, the aggregate revenue of all agents would be maximized when the distance traveled to recover FADs is minimized. We understand that profit maximization is not always ensured and that skippers refuse to share their know-how as FAD harvesters without commensurate reward, which are the main reasons for the failure of this initiative type.

In this context, our research aims to evaluate a fairer sharing method, which also generates more benefits for all and encourages cooperation among the various tuna vessels. Previous research has focused more on the use of FADs and their implications, rather than actually helping to optimize the tuna fleet's fishing activity (Groba et al., 2020; Groba, Sartal, & Vázquez, 2015; 2018). Given the new FAD restrictions, which are likely to tighten further, the tuna fishing industry needs to become more efficient and find ways in which to be more optimal. However, if these practices are to prevail on a day-to-day basis, the industry must be fair with respect to the possible sharing of FADs. Consequently, we have studied the same situation but from a cooperative approach. We associate a cooperative game with each tuna problem, for which the value of the grand coalition is no more than the maximum aggregate revenue of the agents. We then computed the Shapley value of the cooperative game, which allocates the maximum aggregate revenue among the agents. The advantage of the Shapley value is that the allocation among the agents meets some principles of fairness, whereas the Bayesian Nash equilibrium responds to private incentives, for example, in Groba et al. (2020).

2.2. Theoretical model

After describing the tuna problem, we begin from the theoretical model of Groba et al. (2020) to propose our own solution based on cooperative models. As we have made a different analysis, some changes were necessary to adapt to the notation of cooperative games better.

Let $N = \{1, \ldots, n\}$ be the set of tuna vessels, briefly, vessels. We assume that all vessels work for the same firm, which we denote by f.

Let Π_N be the set of all orders over the finite set $N \subset \mathbb{N}$.

There is a finite number of FADs that have been assigned to the vessels. We denote the set of all FADs by $B = \{b_1, \ldots, b_m\}$. The position of each FAD $b \in B$ is given by $(x(b), y(b))$ where $x(b)$ denotes the latitude and $y(b)$ the longitude. In addition, we denote $(x(i), y(i))$ the position of vessel i at the beginning of the process.

In practice, each vessel releases its FADs in the ocean, in specific positions chosen by the skipper based on his experience. Thus, each FAD is initially assigned to the vessel that released the FAD. For each $b \in B$, let $\alpha(b) \in N$ denote the vessel to which FAD b is initially assigned. Thus, each vessel $i \in N$ has an initial endowment of FADs given by

$$B_i^\alpha = \{b \in B : \alpha(b) = i\}.$$

Given $S \subset N$, let $d(S)$ denote the minimum distance that vessels in S have to travel for recovering all FADs in $\bigcup_{i \in S} B_i^\alpha$. We now explain formally how to compute $d(S)$.

Let $\varrho : \bigcup_{i \in S} B_i^\alpha \to S$ a function that reassigns the FADs initially assigned to vessels in S $\left(\bigcup_{i \in S} B_i^\alpha\right)$ among vessels in S. We denote by $R\left(\bigcup_{i \in S} B_i^\alpha\right)$ the set of all possible functions $\varrho : \bigcup_{i \in S} B_i^\alpha \to S$.

For each vessel $i \in S$ we denote by B_i^ϱ the FADs assigned by ϱ to vessel i. Namely,

$$B_i^\varrho = \left\{ b \in \bigcup_{i \in S} B_i^\alpha : \varrho(b) = i \right\}.$$

Given $\pi \in \Pi_{B_i^\varrho}$ we denote by $d^\varrho(i, \pi)$ the distance traveled by vessel i for recovering the FADs in B_i^ϱ following the order given by π. We denote by $d^\varrho(i)$ the minimum distance traveled by vessel i for recovering all FADs in B_i^ϱ, which is computed minimizing $d^\varrho(i, \pi)$ on $\Pi_{B_i^\varrho}$. Formally,

$$d^\varrho(i) = \min \left\{ d^\varrho(i, \pi) : \pi \in \Pi_{B_i^\varrho} \right\}.$$

Let $d^\varrho(S)$ denote the distance traveled by all vessels in S for recovering all FADs in $\bigcup_{i \in S} B_i^\alpha$ when each vessel $i \in S$ recovers the FADs in B_i^ϱ. Thus,

$$d^\varrho(S) = \sum_{i \in S} d^\varrho(i).$$

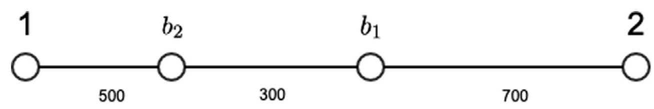

Fig. 1. Example 1.

Then, $d(S)$ is computed by minimizing $d^{\varrho}(S)$ on $R\left(\bigcup_{i\in S} B_i^{\alpha}\right)$. Namely,

$$d(S) = \min\left\{ d^{\varrho}(S) : \varrho \in R\left(\bigcup_{i\in S} B_i^{\alpha}\right)\right\}.$$

Notice that given $S, T \subset N$ with $S \cap T = \varnothing$ we have

$$d(S \cup T) \le d(S) + d(T). \tag{1}$$

To compute the minimum distance in practice is difficult because there is not a polynomial algorithm. In the computations of Section 5, we have used the nearest neighbor strategy.

We make the following assumptions.

1. The firm knows the location of all vessels and all FADs. Each vessel knows the location of all of its assigned FADs and does not know the location of the FADs assigned to other vessels.
2. Each vessel has a cost c per mile traveled, and this cost is paid for by the firm.
3. Vessels cannot know in advance the amount of tuna they will find at each FAD. Using $q(b)$, we denote the amount of tuna recovered in FAD b. This amount will be known only after fishing. Using $q_i^{\alpha} = \sum_{b\in B_i^{\alpha}} q(b)$, we denote the total amount of tuna recovered in the FADs of vessel i.
4. Each vessel receives a price p for each ton of tuna recovered.
5. The firms sells the tuna collected from the vessels. Let p_f be the price for each ton of tuna.

Once a vessel has recovered all of its FADs, the firm pays the vessel. Namely, each vessel $i \in N$ obtains

$$pq_i^{\alpha} \tag{2}$$

The firm sells all the tuna recovered. In addition, the firm has to pay the cost associated with the vessels' travel. Thus, the firm obtains

$$\left(p_f - p\right)\sum_{i\in N} q_i^{\alpha} - c\sum_{i\in N} d(i) \tag{3}$$

We assume that every vessel generates revenues by recovering the tuna of its FADs. Namely, the fuel cost for recovering all the vessels' FADs plus the price paid by the firm to the vessel is not larger than the price in the market of the total amount of tuna recovered. Formally, for each $i \in N$,

$$p_f q_i^{\alpha} \ge pq_i^{\alpha} + cd(i)$$

In the following numerical and simple example, we try to clarify the previously introduced model. It will be used through the paper for clarifying other concepts introduced later.

Example 1. We consider the following tuna vessel fishing problem (Fig. 1).

- $N = \{1, 2\}$.
- $B = \{b_1, b_2\}$.
- The vessels and FADs are located in a line, from left to right. The distance between Vessel 1 and FAD b_2 is 500; the distance between FAD b_2 and b_1 is 300; and the distance between FAD b_1 and Vessel 2 is 700.
- $\alpha(b_1) = 1$ and $\alpha(b_2) = 2$. Thus, $B_1^{\alpha} = \{b_1\}$, and $B_2^{\alpha} = \{b_2\}$.
- $d(1) = 500 + 300$, $d(2) = 700 + 300$, and $d(1, 2) = 500 + 700$ (Vessel 1 recovers b_2, and Vessel 2 recovers b_1).
- $c = 29$.
- $q(b_1) = 110$, $q(b_2) = 130$, $q_1^{\alpha} = 110$, and $q_2^{\alpha} = 130$.
- $p = 140$, and $p_f = 1400$.

As the fuel cost is paid by the firm, vessels do not have a direct incentive to share their FADs to minimize the distance traveled. Nevertheless, the firm has incentives. If the fuel cost is reduced, then the revenue of the firm will be increased. In addition, the aggregated revenue of all agents is maximized when the distance traveled for recovering the FADs is minimized.

Thus, having established the theoretical assumptions and having seen the previous example, we evaluated the previously described tuna fishing vessel problem (Section 2.1), but from a cooperative approach based on the calculation of the Shapley value.

3. The cooperative game approach to the tuna fishing vessel problem

The cooperative game approach to the tuna vessel problem requires several assumptions. First, we assume that the vessels and the firm cooperate and share the revenues of the cooperation among themselves. Second, we take the *status quo*, from where cooperation could arise, as the situation in which vessels recover their FADs and the firm pays them according to the amount of tuna fished. Thus, the revenues come from the vessels' reduction in fuel consumption, which is a result of the FADs reassignments among the vessels. Finally, we associate each tuna fishing vessel problem with a cooperative game. Then, we study the Shapley value of the cooperative game, and we argue that the Shapley value is a good way in which to divide the revenues of the cooperation among the vessels and the firm.

We first introduce the technical notation. We denote the set of real numbers as $\mathbb{R}$. Given a finite set N, we denote the cardinality of N as $|N|$.

Given $x, y \in \mathbb{R}^N$, we write $x \le y$, when $x_i \le y_i$ for all $i \in N$. Given $x \in \mathbb{R}^N$ and $S \subset N$, we denote $x_S = (x_i)_{i\in S}$.

Given $\pi \in \Pi_N$, let $Pre(i, \pi)$ denote the set of elements of N, which come before i in the order given by π, *i. e.* $Pre(i, \pi) = \{j \in N \mid \pi(j) < \pi(i)\}$. Given $S \subset N$, let π_S denote the order induced by π among agents in S.

We now introduce some well-known concepts of cooperative game theory that will be used later.

A game with transferable utility (*TU* game) is a pair $\left(N', v\right)$, where N' is a finite set and $v : 2^{N'} \to \mathbb{R}$ is the *characteristic function* satisfying $v(\emptyset) = 0$.

We denote by $Sh\left(N', v\right)$ the *Shapley value* (Shapley, 1953) of the *TU* game $\left(N', v\right)$. For each $i \in N'$,

$$\begin{aligned} Sh_i\left(N', v\right) &= \frac{1}{|N'|!}\sum_{\pi\in\Pi_{N'}} \left(v(Pre(i,\pi)\cup\{i\}) - v(Pre(i,\pi))\right) \\ &= \sum_{S\subset N'\setminus\{i\}} \frac{|S|!\left(|N'| - |S| - 1\right)!}{|N'|!}\left(v(S\cup\{i\}) - v(S)\right). \end{aligned} \tag{4}$$

When no confusion arises we write v instead of $\left(N', v\right)$.

We now associate a cooperative game $\left(N', v^{q^{\alpha}}\right)$ with every tuna fishing vessel problem as follows. Here, $N' = N \cup \{f\}$, where N is the set of vessels, and f is the firm. Given $i \in N$, $v^{q^{\alpha}}(i)$ is the revenue of vessel i from recovering all of its FADs. Namely, $v^{q^{\alpha}}(i) = pq_i^{\alpha}$. Similarly, $v^{q^{\alpha}}(f)$ is given by expression (3).

Given $S \subset N$, there is no extra revenues from the cooperation. Namely,

$$v^{q^{\alpha}}(S) = \sum_{i\in S} pq_i^{\alpha} = \sum_{i\in S} v^{q^{\alpha}}(i). \tag{5}$$

Table 1
Characteristic function of Example 1.

Coalition	Characteristic function
S	$v^{q^\alpha}(S)$
1	$140 \times 110 = 15,400$
2	$140 \times 130 = 18,200$
f	$(1400 - 140) \times (110 + 130) - 29 \times (800 + 1000) = 250,200$
1,2	33,600
$1, f$	265,600
$2, f$	268,400
$1, 2, f$	301,200

Table 2
Revenues of Example 1.

Agents	1	2	f
$v^{q^\alpha}(i)$	15,400	18,200	250,200
$Sh_i\left(v^{q^\alpha}\right)$	21,200	24,000	256,000

Given $S \subset N$, we define $v^{q^\alpha}(S \cup \{f\})$ as follows. Each vessel in $N \backslash S$ recovers its FADs and sells to the firm. The FADs assigned to S are reassigned among vessels in S, minimizing the total distance traveled. Each vessel in S recovers its reassigned FADs. The fuel of all vessels is paid for the firm. Namely,

$$v^{q^\alpha}(S \cup \{f\}) = \left(p_f - p\right) \sum_{i \in N} q_i^\alpha + p \sum_{i \in S} q_i^\alpha - c\left[d(S) + \sum_{i \in N \backslash S} d(i)\right]$$
$$= \left(p_f - p\right) \sum_{i \in N \backslash S} q_i^\alpha + p_f \sum_{i \in S} q_i^\alpha - c\left[d(S) + \sum_{i \in N \backslash S} d(i)\right].$$

We can give an alternative expression for $v^{q^\alpha}(S \cup \{f\})$ through $v^{q^\alpha}(i)$.

$$v^{q^\alpha}(S \cup \{f\}) = p \sum_{i \in S} q_i^\alpha + \left(p_f - p\right) \sum_{i \in N} q_i^\alpha - c \sum_{i \in N} d(i)$$
$$- c\left[d(S) - \sum_{i \in S} d(i)\right]$$
$$= \sum_{i \in S \cup \{f\}} v^{q^\alpha}(i) + c\left[\sum_{i \in S} d(i) - d(S)\right] \qquad (6)$$

Then, the revenue generated when all agents cooperate is given by

$$v^{q^\alpha}(N \cup \{f\}) = p_f \sum_{i \in N} q_i^\alpha - cd(N)$$
$$= \sum_{i \in N \cup \{f\}} v^{q^\alpha}(i) + c\left(\sum_{i \in N} d(i) - d(N)\right).$$

In Example 1, the characteristic function is given by Table 1.

The first row of Table 2 $\left(v^{q^\alpha}(i)\right)$ is the revenue obtained by the agents when they decide not to cooperate (namely, they do not share the FADs). The second row is the utility achieved when they cooperate, and the revenues are divided following the Shapley value.

Notice that all agents obtain more under the Shapley value. Later, we will prove that this fact holds for any tuna fishing vessel problem. Moreover, the earnings from cooperation $\left(Sh_i\left(v^{q^\alpha}\right) - v^{q^\alpha}(i)\right)$ are the same for all agents (5800). This does not happen in general, for instance, in the case of real data, which we will discuss in Section 5. Nevertheless, we will prove that this happens for any case with two vessels.

In Proposition 1, we prove that the Shapley value provides each vessel and the firm revenues larger than the revenues they will obtain in the case of non-cooperation. We first prove the following lemma.

Lemma 1. *For each tuna fishing vessel problem, v^{q^α} is superadditive. Namely, for each $S, T \subset N \cup \{f\}$ with $S \cap T = \emptyset$, we have that*

$$v^{q^\alpha}(S \cup T) \geq v^{q^\alpha}(S) + v^{q^\alpha}(T).$$

Proof. We consider two cases.

Assume first that $f \notin S \cup T$. By (5),

$$v^{q^\alpha}(S \cup T) = \sum_{i \in S \cup T} v^{q^\alpha}(i) = v^{q^\alpha}(S) + v^{q^\alpha}(T).$$

Assume now that $f \in S \cup T$. We consider that $f \in S$ (the case $f \in T$ is similar and we omit it). By (6)

$$v^{q^\alpha}(S \cup T) = \sum_{i \in S \cup T} v^{q^\alpha}(i) + c\left[\sum_{i \in (S \backslash \{f\}) \cup T} d(i) - d((S \backslash \{f\}) \cup T)\right]$$
$$= \sum_{i \in S} v^{q^\alpha}(i) + c\left[\sum_{i \in S \backslash \{f\}} d(i) - d(S \backslash \{f\})\right] + \sum_{i \in T} v^{q^\alpha}(i)$$
$$+ c\left[d(S \backslash \{f\}) + \sum_{i \in T} d(i) - d((S \backslash \{f\}) \cup T)\right]$$
$$= v^{q^\alpha}(S) + v^{q^\alpha}(T)$$
$$+ c\left[d(S \backslash \{f\}) + \sum_{i \in T} d(i) - d((S \backslash \{f\}) \cup T)\right].$$

By (1),

$$d(S \backslash \{f\}) + \sum_{i \in T} d(i) \geq d(S \backslash \{f\}) + d(T) \geq d((S \backslash \{f\}) \cup T).$$

Hence, $v^{q^\alpha}(S \cup T) \geq v^{q^\alpha}(S) + v^{q^\alpha}(T)$. □

Proposition 1. *For each tuna fishing vessel problem and each $i \in N \cup \{f\}$,*

$$Sh_i\left(v^{q^\alpha}\right) \geq v^{q^\alpha}(i).$$

Proof. By (4), it is enough to prove that for each $i \in N \cup \{f\}$ and each $S \subset (N \cup \{f\}) \backslash \{i\}$, we have that $v^{q^\alpha}(S \cup \{i\}) - v^{q^\alpha}(S) \geq v^{q^\alpha}(i)$. For Lemma 1, it holds. □

Consider the cooperative game v^d defined as follows. For each $S \subset N$,

$$v^d(S) = 0 \text{ and } v^d(S \cup \{f\}) = d(S).$$

Notice that this game is computed using only the distances.

The next proposition gives an expression for the Shapley value of v^{q^α} using the game v^d and the individual value of each agent. We prove that, if we apply the Shapley value, every vessel is paid according to the amount of tuna recovered in its FAD, plus an extra amount that comes from the fuel savings. In addition, the firm is paid according to the amount of tuna recovered in all of the FADs, plus an extra amount that comes from the fuel savings.

Proposition 2. *For each tuna fishing vessel problem, the Shapley value of v^{q^α} can be computed as follows:*

$$Sh_i\left(v^{q^\alpha}\right) = v^{q^\alpha}(i) + c\left[\frac{d(i)}{2} - Sh_i\left(v^d\right)\right] \text{ for each } i \in N$$

$$Sh_f\left(v^{q^\alpha}\right) = v^{q^\alpha}(f) + c\left[\sum_{i \in N} \frac{d(i)}{2} - Sh_f\left(v^d\right)\right].$$

Proof. Consider the cooperative games v^1 and v^2, where for each $S \subset N$,

$$v^1(S) = \sum_{i \in S} v^{q^\alpha}(i) \text{ and } v^1(S \cup \{f\}) = \sum_{i \in S \cup \{f\}} v^{q^\alpha}(i).$$

$$v^2(S) = 0 \text{ and } v^2(S \cup \{f\}) = \sum_{i \in S} d(i).$$

By (5) and (6), we have that for each $S \subset N \cup \{f\}$,

$$v^{q^\alpha}(S) = v^1(S) + c\left[v^2(S) - v^d(S)\right].$$

As the Shapley value is a linear function on the characteristic function, we have that for each $i \in N \cup \{f\}$,

$$Sh_i\left(v^{q^\alpha}\right) = Sh_i\left(v^1\right) + c\left[Sh_i\left(v^2\right) - Sh_i\left(v^d\right)\right].$$

By (4), for each $i \in N \cup \{f\}$,

$$Sh_i\left(v^1\right) = v^{q^\alpha}(i).$$

We now compute $Sh_i\left(v^2\right)$ for each $i \in N \cup \{f\}$. Let π be an order over $N \cup \{f\}$. Given $i \in N$,

$$v^2(Pre(i,\pi) \cup \{i\}) - v^2(Pre(i,\pi)) = \begin{cases} d(i) & \text{iff} \in Pre(i,\pi) \\ 0 & \text{otherwise.} \end{cases}$$

As $f \in Pre(i,\pi)$ in half of the orders,

$$Sh_i\left(v^2\right) = \frac{d(i)}{2}.$$

Because the Shapley value satisfies efficiency (namely, $\sum_{i \in N \cup \{f\}} Sh_i(v^2) = v^2(N \cup \{f\})$ and for each $i \in N$, $Sh_i\left(v^2\right) = \frac{d(i)}{2}$, we deduce that

$$Sh_f\left(v^2\right) = \sum_{i \in N} \frac{d(i)}{2}.$$

□

As observed, another important consequence of this proposition is related to the monotonicity property of the Shapley value. If the value of all coalitions involving an agent increases, then the Shapley value of this agent increases. Nevertheless, it could be the case that the Shapley value of the rest of the agents decrease. Then, the improvement of an agent's performance can produce a negative externality among the rest of the agents. In the case of tuna fishing vessel problems, we can study a similar aspect. Suppose that the catches in the FADs of a vessel increase, what happens? According to Proposition 2, the Shapley value of the vessel and the firm increase, whereas the Shapley values of the rest of the vessels remain the same. Thus, the improvement of the performance of a vessel produces a positive externality in the firm and nothing in the rest of the vessels.

Finally, in Proposition 3, we prove that, for the two-vessel case, the revenues of cooperation under the Shapley value are the same for all agents.

Proposition 3. *For each tuna fishing vessel problem where* $N = \{1,2\}$, *we have*

$$Sh_1\left(v^{q^\alpha}\right) - v^{q^\alpha}(1) = Sh_2\left(v^{q^\alpha}\right) - v^{q^\alpha}(2) = Sh_f\left(v^{q^\alpha}\right) - v^{q^\alpha}(f).$$

Proof. By Proposition 2,

$$Sh_1\left(v^{q^\alpha}\right) - v^{q^\alpha}(1) = c\left[\frac{d(1)}{2} - Sh_1\left(v^d\right)\right]$$

$$Sh_2\left(v^{q^\alpha}\right) - v^{q^\alpha}(2) = c\left[\frac{d(2)}{2} - Sh_2\left(v^d\right)\right], \text{ and}$$

$$Sh_f\left(v^{q^\alpha}\right) - v^{q^\alpha}(f) = c\left[\frac{d(1)}{2} + \frac{d(2)}{2} - Sh_f\left(v^d\right)\right]$$

By (4),

$$Sh_1\left(v^{q^\alpha}\right) - v^{q^\alpha}(1) = c\left[\frac{d(1)}{2} - \frac{2d(1,2) + d(1) - 2d(2)}{6}\right] = \frac{c}{3}[d(1) + d(2) - d(1,2)].$$

Similarly, we can prove that

$$Sh_2\left(v^{q^\alpha}\right) - v^{q^\alpha}(2) = \frac{c}{3}[d(1) + d(2) - d(1,2)].$$

Finally, by (4)

$$Sh_f\left(v^{q^\alpha}\right) - v^{q^\alpha}(f) = c\left[\frac{d(1)}{2} + \frac{d(2)}{2} - \frac{2d(1,2) + d(1) + d(2)}{6}\right] = \frac{c}{3}[d(1) + d(2) - d(1,2)].$$

□

4. Implementing the Shapley value

In this section, we explain how to implement in practice the allocation suggested by the Shapley value. Usually, to achieve the allocation proposed by the Shapley value, agents need to bargain among them. Sometimes, the bargaining could be complicated because each agent tries to maximize its own allocation. In this case, we do not have this problem because the allocation could be obtained through a procedure in which no direct agreement among agents is needed. Each agent takes a decision involving itself, and if all agents decide rationally, then the allocation obtained through the mechanism is the Shapley value.

We associate a non-cooperative game in extensive form with each tuna fishing vessel problem. Thus, we study the Subgame Perfect Nash Equilibria ($SPNE$) of the non-cooperative game. Our main result says that the Shapley value of the cooperative game can be obtained as the payoff vector associated with a $SPNE$ of the non-cooperative game.

We now introduce the non-cooperative game formally.

1. **Stage 1. Sharing information about the FADs**.
   Each vessel independently decides if it share its FADs with other vessels. Formally, the action set of each vessel i is given by

   $$A_i^1 = \{yes, no\}.$$

   Then, *yes* means that vessel i share its FADs, and *no* means that vessel i does not share its FADs.
   Let $a_i^1 \in A_i^1$ denote the action chosen by vessel i. We denote that

   $$N^{yes} = \left\{i \in N : a_i^1 = yes\right\} \text{ and } N^{no} = \left\{i \in N : a_i^1 = no\right\}.$$

2. **Stage 2. Reassigning the FADs**.
   The firm reassigns the FADs among the vessels that decided to share it. The other vessels maintain the initial assignment.
   Formally, the action set of the firm is given by

   $$A_f^2 = \left\{ \begin{array}{c} \varrho : B \to N : \varrho(b) = \alpha(b) \text{ if } b \in \bigcup_{i \in N^{no}} B_i^\alpha \\ \text{and } \varrho(b) \in N^{yes} \text{ if } b \in \bigcup_{i \in N^{yes}} B_i^\alpha \end{array} \right\}$$

   Thus, each vessel i is reassigned to the following FADs:

   $$B_i^\varrho = \{b \in B : \varrho(b) = i\}.$$

   Notice that for each $i \in N^{no}$, $B_i^\varrho = B_i^\alpha$.
   Each vessel $i \in N^{yes}$ recovers the FADs in B_i^ϱ travelling the minimum distance. Let $d^\varrho(N^{yes})$ denote the sum of the distances traveled by all vessels in N^{yes}.

3. **Stage 3. Fishing**.
The vessel fish in the new assigned FADs. For each $i \in N$ and each $b \in B_i^{\varrho}$, we denote by $q^*(b)$ the amount of tuna in FAD b. Remember that $q(b)$ denotes the amount of tuna recovered by vessel i in FAD b. Of course, $q(b) \leq q^*(b)$. At each FAD b, vessel i can fish efficiently and recover all tuna in the FAD (namely, $q(b) = q^*(b)$), or not to fish efficiently and recover only a part of the tuna (namely, $q(b) < q^*(b)$). Thus, the action set of vessel i is given by
$$A_i^3 = \left\{ (q(b))_{b \in B_i^{\varrho}} : 0 \leq q(b) \leq q^*(b) \text{ for all } b \in B_i^{\varrho} \right\}.$$
4. **Stage 4. The firm pays the vessels**.
Vessels that decided not to share their FADs are paid according to the amounts of tuna recovered. Vessels that decided to share their FADs are paid according with the Shapley value of the cooperative game induced by N^{yes}. Formally,
Each $i \in N^{no}$ receives pq_i^{α}.
Each $i \in N^{yes}$ receives $Sh_i\left(N^{yes} \cup \{f\}, v^{q^{\alpha},\varrho}\right)$, where $v^{q^{\alpha},\varrho}$ is defined as follows. For each $S \subset N^{yes}$,
   - $v^{q^{\alpha},\varrho}(S)$ is defined as in (5).
   - $v^{q^{\alpha},\varrho}(S \cup \{f\})$ is defined as in (6) by replacing $d(S)$ by $d^{\varrho}(S)$, where $d^{\varrho}(N^{yes})$ has been defined at Stage 2 and $d^{\varrho}(S) = d(S)$ when $S \neq N^{yes}$.

The firm sells all of the tuna recovered, pays the cost of the fuel, and pays to the vessels as above. Thus, the firm receives
$$p_f \sum_{i \in N} q_i^{\alpha} - c\left(\sum_{i \in N^{no}} d(i) + d^{\varrho}(N^{yes}) \right) - \sum_{i \in N^{no}} pq_i^{\alpha} - \sum_{i \in N^{yes}} Sh_i\left(N^{yes} \cup \{f\}, v^{q^{\alpha},\varrho}\right). \quad (7)$$

The first and most important result indicates that if vessels share their FADs, the firm reassigns the FADs, minimizing the distance travelled, and the vessels fish efficiently, then, we have a *SPNE* of the non-cooperative game. Under this *SPNE*, the payoff to every agent (the vessels and the firm) coincides with the Shapley value of the cooperative game.

Proposition 4. *The following combination of strategies is a SPNE* :

Stage 1. For each $i \in N$, $a_i^1 = yes$.
Stage 2. The firm selects ϱ *such that* $\varrho(b) = \alpha(b)$ *if* $b \in \bigcup_{i \in N^{no}} B_i^{\alpha}$ *and* $d^{\varrho}(N^{yes}) = d(N^{yes})$.
Stage 3. For each $i \in N$, *each* ϱ *selected by the firm at Stage 2, and each* $b \in B_i^{\varrho}$, $q(b) = q^*(b)$.

Besides, the payoff allocation induced by the combination of strategies is the Shapley value of the cooperative game.

Proof. It is obvious that the payoff allocation induced by the combination of strategies is the Shapley value of the cooperative game.

We now prove that this combination is a *SPNE*. This game has three subgames, given by the first three stages. We proceed with backward induction. Then, we first analyze the subgame given by Stage 3. Let $i \in N$. Suppose that vessel i, instead of fishing efficiently ($q(b) = q^*(b)$ for each $b \in B_i^{\varrho}$), decides to do something different. Namely, vessel i plays $\left(q'(b)\right)_{b \in B_i^{\varrho}} \in A_i^3$ such that $q'(b) < q^*(b)$ for some $b \in B_i^{\varrho}$. Hence,
$$q_i'^{\varrho} = \sum_{b \in B_i^{\varrho}} q'(b) < \sum_{b \in B_i^{\varrho}} q^*(b) = q_i^{\varrho}.$$

We consider two cases.

1. $i \in N^{no}$. Playing $(q^*(b))_{b \in B_i^{\varrho}}$ vessel i yields pq_i^{ϱ}, whereas playing $\left(q'(b)\right)_{b \in B_i^{\varrho}}$ generates $pq_i'^{\varrho}$, which is smaller. Thus, vessel i does not improve by deviating from the strategy of the statement at Stage 3.
2. $i \in N^{yes}$. By playing $(q^*(b))_{b \in B_i^{\varrho}}$, and using arguments similar to those used in the proof of Proposition 2, we can prove that vessel i yields
$$Sh_i\left(N^{yes} \cup \{f\}, v^{q^{\alpha},\varrho}\right) = p \sum_{b \in B_i^{\alpha}} q^*(b) + c\left[\frac{d(i)}{2} - Sh_i\left(v^{d^{\varrho}}\right) \right].$$
By playing $\left(q'(b)\right)_{b \in B_i^{\varrho}}$ and using arguments similar to those used in the proof of Proposition 2, we can prove that vessel i yields
$$p\left(\sum_{b \in B_i^{\alpha} \cap B_i^{\varrho}} q'(b) + \sum_{b \in B_i^{\alpha} \setminus B_i^{\varrho}} q^*(b) \right) + c\left[\frac{d(i)}{2} - Sh_i\left(v^{d^{\varrho}}\right) \right].$$
Because $q'(b) \leq q^*(b)$ for all $b \in B_i^{\alpha} \cap B_i^{\varrho}$, vessel i does not improve by deviating from the strategy of the statement at Stage 3.

We now analyze the subgame given by Stage 2. Suppose that the firm chooses ϱ' such that $d^{\varrho'}(N^{yes}) > d(N^{yes})$.

By (7), the utility of the firm under ϱ minus the utility of the firm under ϱ' is the following:
$$c\left(d^{\varrho'}(N^{yes}) - d^{\varrho}(N^{yes})\right) + \sum_{i \in N^{yes}} Sh_i\left(N^{yes} \cup \{f\}, v^{q^{\alpha},\varrho'}\right) - \sum_{i \in N^{yes}} Sh_i\left(N^{yes} \cup \{f\}, v^{q^{\alpha},\varrho}\right). \quad (8)$$

Similarly to Proposition 2, we can prove that
$$\sum_{i \in N^{yes}} Sh_i\left(N^{yes} \cup \{f\}, v^{q^{\alpha},\varrho'}\right) = \sum_{i \in N^{yes}} \left(pq_i^{\alpha} + c\left[\frac{d(i)}{2} - Sh_i\left(N^{yes} \cup \{f\}, v^{d^{\varrho'}}\right) \right] \right) \text{ and}$$
$$\sum_{i \in N^{yes}} Sh_i\left(N^{yes} \cup \{f\}, v^{q^{\alpha},\varrho}\right) = \sum_{i \in N^{yes}} \left(pq_i^{\alpha} + c\left[\frac{d(i)}{2} - Sh_i\left(N^{yes} \cup \{f\}, v^{d^{\varrho}}\right) \right] \right).$$

Thus, (8) coincides with
$$\begin{aligned} & c\left(d^{\varrho'}(N^{yes}) - d^{\varrho}(N^{yes})\right) \\ & + c \sum_{i \in N^{yes}} \left(Sh_i\left(N^{yes} \cup \{f\}, v^{d^{\varrho}}\right) - Sh_i\left(N^{yes} \cup \{f\}, v^{d^{\varrho'}}\right) \right) \\ & = c\left(d^{\varrho'}(N^{yes}) - d^{\varrho}(N^{yes})\right) \\ & + c\left(v^{d^{\varrho}}(N^{yes} \cup \{f\}) - Sh_f\left(N^{yes} \cup \{f\}, v^{d^{\varrho}}\right) \right. \\ & \left. - v^{d^{\varrho'}}(N^{yes} \cup \{f\}) + Sh_f\left(N^{yes} \cup \{f\}, v^{d^{\varrho'}}\right) \right) \\ & = c\left(Sh_f\left(N^{yes} \cup \{f\}, v^{d^{\varrho'}}\right) - Sh_f\left(N^{yes} \cup \{f\}, v^{d^{\varrho}}\right) \right). \end{aligned}$$

Because the Shapley value is an additive on the characteristic function,
$$c\left(Sh_f\left(N^{yes} \cup \{f\}, v^{d^{\varrho'}}\right) - Sh_f\left(N^{yes} \cup \{f\}, v^{d^{\varrho}}\right) \right) = c\left(Sh_f\left(N^{yes} \cup \{f\}, v^{d^{\varrho'}} - v^{d^{\varrho}}\right) \right).$$

Notice that
$$v^{d^{\varrho'}}(S) - v^{d^{\varrho}}(S) = \begin{cases} d^{\varrho'}(N^{yes}) - d(N^{yes}) & \text{if} S = N^{yes} \cup \{f\} \\ 0 & \text{if} S \neq N^{yes} \cup \{f\}. \end{cases}$$

Thus, $Sh_f\left(N^{yes} \cup \{f\}, v^{d^{\varrho'}} - v^{d^{\varrho}}\right) \geq 0$. Hence, the firm does not improve by deviating from the strategy of the statement at Stage 2.

We now analyze the subgame given by Stage 1. Let $i \in N$. Playing *yes*, vessel i yields $Sh_i\left(N \cup \{f\}, v^{q^{\alpha}}\right)$. Suppose that vessel i plays *no* instead of *yes*. Then $N^{no} = \{i\}$ and $N^{yes} = N \backslash \{i\}$. Thus, vessel i yields pq_i^{α}. By Proposition 1, vessel i does not improve by deviating from the strategy of the statement at Stage 1. □

The non-cooperative game has more *SPNE*. In the next proposition, we prove that if vessels do not share their FADs, the firms reassign the FADs, minimizing the distance traveled, and vessels fish efficiently. Then we have a *SPNE*.

Proposition 5. *The following combination of strategies is a SPNE :*

Stage 1. For each $i \in N$, $a_i^1 = no$.
Stage 2. The firm selects ϱ such that $\varrho(b) = \alpha(b)$ if $b \in \bigcup_{i \in N^{no}} B_i^{\alpha}$ and $d^{\varrho}(N^{yes}) = d(N^{yes})$.
Stage 3. For each $i \in N$, each ϱ is selected by the firm at Stage 2, and each $b \in B_i^{\varrho}$, $q(b) = q^*(b)$.

The payoff allocation induced by the combination of strategies is $\left(v^{q^{\alpha}}(i)\right)_{i \in N \cup \{f\}}$, the vector of the individual values of the cooperative game.

Proof. It is obvious that the payoff allocation induced by the combination of strategies is $\left(v^{q^{\alpha}}(i)\right)_{i \in N \cup \{f\}}$.

Notice that the strategies of Stages 2 and 3 coincide with the ones of the statement of Proposition 4. Thus, in the subgames of Stages 2 and 3, we have a *NE*.

We now analyze the subgame given by Stage 1. Let $i \in N$. Playing *no* vessel i obtains pq_i^{α}. Suppose that vessel i plays *yes* instead of *no*. Then $N^{yes} = \{i\}$. Thus, vessel i obtains $Sh_i\left(\{i, f\}, v^{q^{\alpha}}\right)$. Using arguments similar to those used in the proof of Proposition 2, we can prove that

$$Sh_i\left(\{i, f\}, v^{q^{\alpha}}\right) = pq_i^{\alpha} + c\left[\frac{d(i)}{2} - Sh_i\left(\{i, f\}, v^{d}\right)\right] = pq_i^{\alpha}.$$

Thus, vessel i does not improve by deviating from the strategy of the statement at Stage 1. □

Propositions 4 and 5 describe, for any tuna fishing vessels problem, two *SPNEs* of the non-cooperative game. Nevertheless, depending on the position of the FADs and initial assignment α, some tuna fishing vessel problems could have more *SPNEs*.

In the next proposition, we give two properties of the *SPNE* described in Proposition 4, that make this *SPNE* more appealing.

Proposition 6.

(a) *Each agent obtains under the SPNE of Proposition 4 at least the same as that under the SPNE of Proposition 5.*
(b) *Suppose that at Stage 3, all vessels fish efficiently in their assigned FADs. Then, in any SPNE the firm select ϱ, minimizing the distances. Besides, in any SPNE, for each vessel to play yes is at least as good as it is to play no.*

Proof.

(a) It is a consequence of Proposition 1
(b) Using arguments similar to those used in the proof of Proposition 4 we can prove that in any *SPNE*, the firm selects ϱ, minimizing the distances.

Let $\left(a_i^1\right)_{i \in N}$ be the action of vessels in Stage 1 in some *SPNEs*. Suppose now that vessel j is playing *no* and decides to play *yes*. Formally, let $\left(a_i'^1\right)_{i \in N}$ and $j \in N$ be such that $a_j^1 = no$, $a_j'^1 = yes$ and $a_i^1 = a_i'^1$ for each $i \in N \backslash j$. Then,

$$N'^{yes} = \left\{i \in N : a_i'^1 = yes\right\} = N^{yes} \cup \{j\} \text{ and}$$
$$N'^{no} = \left\{i \in N : a_i^1 = no\right\} = N^{no} \backslash \{j\}.$$

We should prove that vessel j playing *yes* yields at least the same result that playing *no* does. By playing *no*, vessel j yields pq_j^{α}. By playing *yes*, vessel j yields $Sh_j\left(N'^{yes} \cup f, v^{q^{\alpha},\varrho}\right)$. Similarly to the proof of Proposition 4, we can prove that

$$Sh_j\left(N'^{yes} \cup f, v^{q^{\alpha},\varrho}\right) \geq pq_j^{\alpha}.$$

□

A *SPNE* predicts the behavior of rational agents. When there is a unique *SPNE*, we can predict how rational agents will behave. Nevertheless, when there are several *SPNEs*, we should compare all of them. Proposition 6 present two properties of the *SPNE* described in Proposition 4 that make it more appealing.

Part (a) says that all vessels and the firm obtain at least the same results under the *SPNE* of Proposition 4 and the *SPNE* of Proposition 5. Thus, it should be relatively easily to coordinate all agents to play the *SPNE* given by Proposition. For instance, the firm could say to the vessels that if all play *yes* at Stage 1, then the revenues will be at least the same as that generated when playing *no*.

Part (b) says that if all vessels fish efficiently at Stage 3, and if vessels play rationally, they should choose *yes* at Stage 1. Notice that when choosing *yes*, every vessel obtains at least the same result as that associated with choosing *no*.

Let us discuss the incentives for fishing efficiently at Stage 3. Each vessel should fish in two types of FADs: the ones assigned initially to the vessel and the ones assigned initially to other vessels but reassigned at Stage 2 to this vessel. In the FADs assigned initially, each vessel has incentives to fish efficiently. Otherwise, at Stage 4, the amount received by the vessel would be smaller. In the case of reassigned FADs, things are different. The amount received by the vessel at Stage 4 does not depend on the amount recovered in those FADs. Thus, fishing efficiently produces the same allocation than fishing not efficiently. We think this aspect is a shortcoming from a theoretical point of view. Nevertheless, we think that in most of the practical cases, this is not the case. Usually, the number of vessels is small (two or three). They face the same situation (with the same vessels and the same firm) for several seasons. This means that if a vessel does not fish efficiently, it will be discovered and could be removed from the fishing group. Thus, we think that the assumption made in part (b) is quite realistic from a practical point of view.

5. Data and results

In this section, we design an experiment that, using real FAD movement data from different tuna fishing firms, empirically evaluates the theoretical propositions described in the previous section. All of this allows us to support the proposed solution through the Shapley value, a much more equitable technique than others used (e.g., Bayesian Nash equilibria) to model this type of situation. One of the main problems in cooperative game theory is how to distribute the total revenue among all players (in this case, the vessels and the firm) equally, but taking into account the contribution of each player. With this idea in mind, we calculate the revenues (in dollars) earned by the vessels and the firm in three scenarios: (i) no FAD sharing, (ii) FAD sharing, and (iii) FAD sharing following the Shapley value. The data sample, provided by Marine Instruments for scientific purposes only, is composed of anonymized real data from three tuna vessels fishing in the eastern Indian Ocean

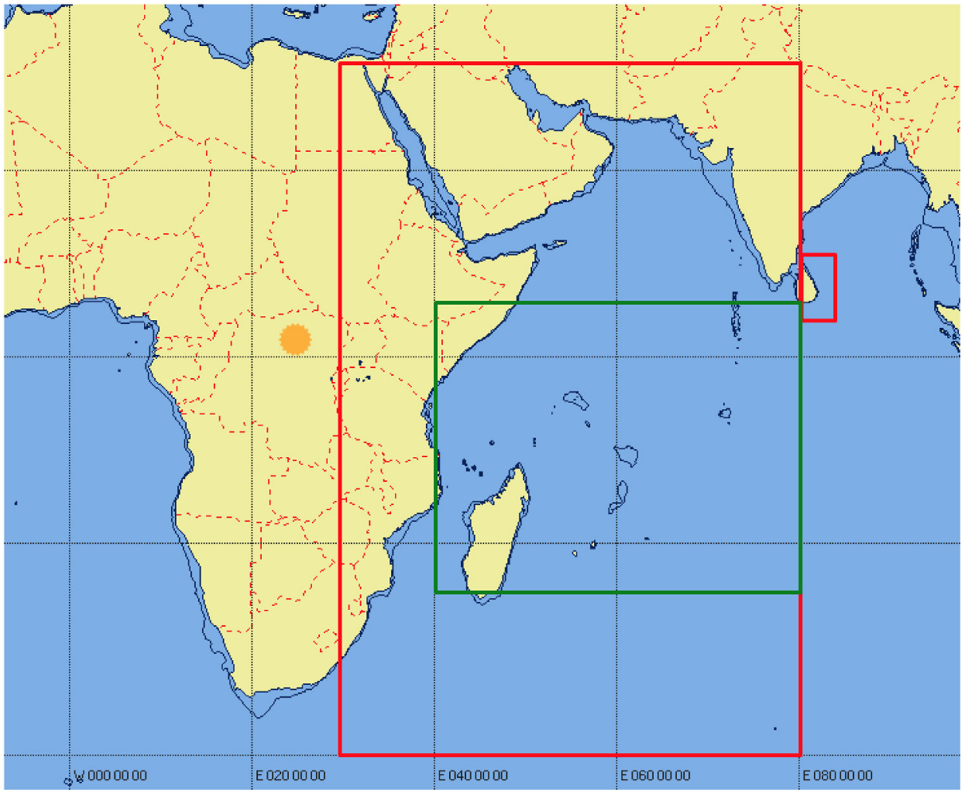

Fig. 2. FAO area no. 51 (in red) and the data area of our sample (in green). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

from April 9 to 23, 2017 in FAO (Food and Agriculture Organization of the United Nations) catch area no. 51 (red rectangle in Fig. 2).

It is worth highlighting three key issues concerning the choice of the sample, the study period, and the dates chosen. Regarding the sample, our data come from a tuna fishing firm composed of three vessels (i.e., three skippers) with 20 FADs per vessel. The decision to use three vessels has a double objective. First, this type of grouping is one of the most common ways of working with tuna vessels (Groba et al., 2020). Second, it was the simplest, and most parsimonious, way to approach our objective and test the three proposed propositions. As for the study period, we considered one month of study because it is a common fishing period for this type of vessel and technique (Groba et al., 2015; 2018). Finally, although the time of year is important in the case of free fishing, there are no differences in the case of FAD use that are relevant from a seasonal point of view (Fonteneau, Chassot, & Bodin, 2013; Maufroy et al., 2016). Thus, this randomly collected information was recorded using Marine Instruments MSB software, a FAD data reception and visualization platform developed by Marine Instruments. In addition, a number of simplifications were taken into account based on Marine Instrument's historical records for vessels working in this area (Table 3).

Table 3
Experiment assumptions.

Description	Value
Number of vessels	3
FADs per vessel	20
Vessel speed	15 knots
Fishing time	3 hours
Cost per ton	$1400
Fuel cost per mile	$29
Skipper benefit	10%
Tons beneath each standard FAD	6
Tons beneath each premium FAD	8

To highlight some of the most salient ones, (i) we assume that tuna vessels sail at 15 knots with an average cost of $29/nautical mile, and (ii) the skipper knows the number of FADs to which he/she has been assigned (initial assignment) but cannot know in advance the final assignment, nor the amount of tuna available in each FAD. Following authors such as Groba et al. (2020), we consider that the average expected amount of tuna from each FAD is approximately six tons. Furthermore, for a correct interpretation of the results -although this value may be slightly different from one firm to another- we have assumed an average profit for skippers of 10% of the total amount of tuna caught (Groba et al., 2020). For the sake of simplicity, we have not paid attention to other fixed costs of the firm, such as crew costs, supplies, fishing licenses, and so forth.

Although more sophisticated techniques exist, we use the nearest neighbor (NN) strategy to retrieve FADs during the simulations in our experiments (Fig. 3). This implies that the final distribution of the FADs was based on the efficient assignment of the nearest FADs. The choice of this method is based on the fact that it is a fast and robust assignment method, commonly used by the tuna industry today and perfectly valid to test our theoretical assumptions. In fact, in the case of using a more efficient recovery strategy, for example, one that takes into account the dynamic nature of drifting FADs, this would imply better results and, therefore, higher profits even for the firm (and therefore for each skipper) due to the optimization of the FAD collection route (Groba et al., 2020).

Table 4 summarizes the real case we have analyzed, in which the 60 FADs owned by three tuna vessels (initial allocation) are optimally shared and reallocated (final allocation) to minimize the total distance traveled together. This reallocation of FADs implies that sometimes a vessel is reassigned 20 FADs (the same amount

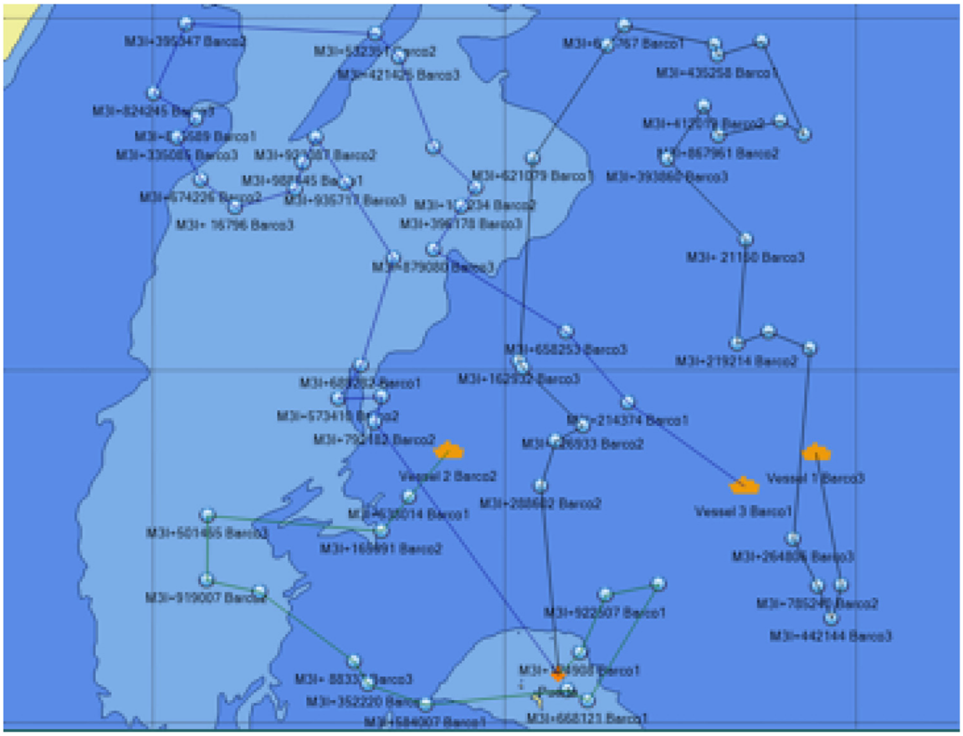

Fig. 3. Example of collecting FADs with NN strategy with 3 vessels.

it had initially), but at other times, it receives more or fewer FADs than it had initially. This is represented graphically with different colors in the columns of Vessel 1, Vessel 2, and Vessel 3.

Knowing both situations, the distance traveled by each vessel is calculated (initial distance), as well as the total distance traveled by the three vessels (initial total distance). In parallel, the MSB software calculates the total distance traveled as a whole thanks to the optimal reallocation (optimal sharing in Table 4), in this case 4139 nm. With these data, and taking into account the expected tons per FAD as well as the tuna and fuel costs per mile (Table 3), we calculate the revenue for each vessel and the firm.

In addition, assuming an expected average quantity of six tons per FAD and taking into account the distances saved, along with the redistribution of the FADs (Table 4), we can empirically validate the Shapley value for revenue sharing between the players (i.e., the vessels and the firm). Moreover, to show the best behavior of the Shapley value, that is its higher fairness taking into account the contribution of each player, we compare it with the results of Groba et al. (2020), who used non-cooperative mechanisms of FAD reallocation based on Bayesian Nash equilibria (Table 5). First, we calculated vessel and firm revenues (in dollars) when FADs are not shared (Table 5 - Revenues without cooperation). Second, we use Groba et al. (2020)'s results as an example of a mechanism that incentivizes vessels to share FADs. In their research, they used Bayesian Nash equilibria of the non-cooperative game to guarantee each vessel at least the same revenue as in the case of not sharing their FADs. We calculate the revenue given by this mechanism, the total improvement, and the cooperative improvement (with respect to non-sharing) to compare them with our proposal. Finally, we do the same, but we consider the Shapley value.

As we have argued in the theoretical model, all agents (i.e., tuna vessels and firm) obtain more revenue by cooperating according to the Shapley value. Through cooperation, each vessel obtains an extra revenue of approximately $19,000 on average, and the firm gains extra revenue totaling approximately $34,000. Comparing the final distances and amounts with the initial ones (as shown in Table 6), it can already be seen that the vessels and the firm gain significant benefits, reaching improvements of more than 100% for the vessels and 14% for the firm. This is due to the fuel saved through the intelligent reallocation of FADs.

Therefore, our empirical results corroborate the findings reached by authors such as Groba et al. (2020), but our work demonstrates how the Shapley value is a more equitable method for two reasons. On the one hand, although the firm loses a part of its improvement (21 points, from 35% to 14%) with respect to the Groba et al. (2020) mechanism, the improvement for tuna vessels is much higher (about 100 points more) for all vessels. On the other hand, with the Shapley value, no vessel loses the opportunity to improve its income. Although it is true that some vessels may have higher profits than others do based on the reallocated FADs (e.g., Vessel 1 at 109% versus Vessel 2 at 117%), the tension that this situation may generate is not at all comparable to other sharing methods such as that in Groba et al. (2020), where some vessels may not benefit from sharing. This may be a key factor for tuna vessel owners to implement this type of solution once and for all, something that is not happening so far despite the fact that these methods are already known in the sector.

To avoid supporting our results on a single experiment, we perform 10 simulations varying the positions of the FADs and the initial position of the vessels. The rest of the parameters are given by Table 3, in which we assume standard FADs (six tons). As in the previous case, we calculate (a) the total improvement by cooperation in each experiment (Table 6 part [a]) and the percentage improvement by cooperation (Table 6 part [b]). As before, the data are from three tuna vessels in the eastern Indian Ocean, specifically in FAO catch area no. 51 (Fig. 2). As can be seen, the results do not differ significantly from the particular case analyzed. All vessels increase their incomes by approximately $19,000, which is an improvement of more than 100% compared with the no-sharing situation. Moreover, the average of the experiments describes a 13.3% improvement in the owner firm's revenue (previously 14%). Therefore, the replication of the cases not only validates the results in

Table 4
Example of the calculations performed in each experiment.

Concept	Vessel1	Vessel2	Vessel3
FADs (initial assignment)	FAD 1.1	FAD 2.1	FAD 3.1
	FAD 1.2	FAD 2.2	FAD 3.2
	FAD 1.3	FAD 2.3	FAD 3.3
	FAD 1.4	FAD 2.4	FAD 3.4
	FAD 1.5	FAD 2.5	FAD 3.5
	FAD 1.6	FAD 2.6	FAD 3.6
	FAD 1.7	FAD 2.7	FAD 3.7
	FAD 1.8	FAD 2.8	FAD 3.8
	FAD 1.9	FAD 2.9	FAD 3.9
	FAD 1.10	FAD 2.10	FAD 3.10
	FAD 1.11	FAD 2.11	FAD 3.11
	FAD 1.12	FAD 2.12	FAD 3.12
	FAD 1.13	FAD 2.13	FAD 3.13
	FAD 1.14	FAD 2.14	FAD 3.14
	FAD 1.15	FAD 2.15	FAD 3.15
	FAD 1.16	FAD 2.16	FAD 3.16
	FAD 1.17	FAD 2.17	FAD 3.17
	FAD 1.18	FAD 2.18	FAD 3.18
	FAD 1.19	FAD 2.19	FAD 3.19
	FAD 1.20	FAD 2.20	FAD 3.20
Initial Distance (nm)	2344	2481	2442
Initial Total Distance (nm)	7268		
Optimal single (nm)	1613	897	1630
Optimal sharing (nm)	4139		
FADs (final assignment)	FAD 1.1	FAD 1.6	FAD 1.9
	FAD 1.2	FAD 1.7	FAD 1.10
	FAD 1.3	FAD 1.8	FAD 1.11
	FAD 1.4	FAD 2.1	FAD 1.12
	FAD 1.5	FAD 2.2	FAD 2.13
	FAD 1.13	FAD 2.3	FAD 2.14
	FAD 1.14	FAD 2.4	FAD 2.15
	FAD 1.15	FAD 3.3	FAD 2.16
	FAD 1.16	FAD 3.4	FAD 2.17
	FAD 1.17	FAD 3.5	FAD 2.18
	FAD 1.18	FAD 3.6	FAD 2.19
	FAD 1.19	FAD 3.7	FAD 2.20
	FAD 1.20	FAD 3.8	FAD 3.9
	FAD 2.5		FAD 3.10
	FAD 2.6		FAD 3.11
	FAD 2.7		FAD 3.12
	FAD 2.8		FAD 3.13
	FAD 2.9		FAD 3.14
	FAD 2.10		FAD 3.15
	FAD 2.11		FAD 3.16
	FAD 2.12		FAD 3.17
	FAD 3.1		FAD 3.18
	FAD 3.2		FAD 3.19
			FAD 3.20
Tons caught initial assigment	120	120	120
Tons caught final assigment	138	78	144

Table 5
Revenue ($) comparison among no sharing, sharing, and sharing using the Shapley value.

		Vessel 1	Vessel 2	Vessel 3	Firm
No sharing FADs (without cooperation)		16,800.0	16,800.0	16,800.0	242,837.2
Sharing FADs (Groba et al., 2020)	Revenue	19,320.0	16,800.0	20,160.0	327,687.5
	Total improvement	2,520.0	0.0	3,360.0	84,850.0
	Improvement cooperation (%)	**15%**	**0%**	**20%**	**35%**
Sharing FADs with Shapley Value	Revenue	35,059.9	36,453.9	35,607.9	276,845.8
	Total improvement	18,259.9	19,653.9	18,807.9	34,008.6
	Improvement cooperation (%)	**109%**	**117%**	**112%**	**14%**

Table 6
a) Total improvement by cooperation, and b) Percentage improvement by cooperation.

	Vessel 1	Vessel 2	Vessel 3	Firm
1	$ 18.259,90	$ 19.653,90	$ 18.807,90	$ 34.008,62
2	$ 19.776,22	$ 21.421,40	$ 21.071,78	$ 36.692,28
3	$ 16.719,68	$ 17.923,94	$ 18.437,99	$ 31.618,14
4	$ 19.339,56	$ 23.222,32	$ 23.512,86	$ 38.602,75
5	$ 18.107,85	$ 18.474,84	$ 19.742,08	$ 34.045,23
6	$ 20.012,33	$ 22.521,45	$ 22.017,07	$ 37.733,39
7	$ 17.445,70	$ 19.415,83	$ 17.971,43	$ 33.161,84
8	$ 19.628,16	$ 20.699,05	$ 19.819,74	$ 35.588,30
9	$ 16.234,54	$ 20.391,67	$ 18.510,44	$ 32.454,88
10	$ 17.839,34	$ 21.922,83	$ 19.703,86	$ 34.990,91
Avg.	$ 18.336,33	$ 20.564,72	$ 19.959,52	$ 34.889,63

	Vessel 1	Vessel 2	Vessel 3	Firm
1	108,7%	117,0%	112,0%	14,0%
2	117,7%	127,5%	125,4%	15,3%
3	74,6%	106,7%	109,7%	10,6%
4	115,1%	103,7%	140,0%	13,1%
5	107,8%	110,0%	117,5%	14,1%
6	89,3%	134,1%	131,1%	12,9%
7	103,8%	115,6%	107,0%	13,3%
8	116,8%	123,2%	118,0%	14,7%
9	72,5%	121,4%	110,2%	10,7%
10	106,2%	130,5%	117,3%	14,1%
Avg.	101,3%	119,0%	118,8%	13,3%

Table 7
Revenue ($) comparison sharing “PREMIUM” FADs with Shapley value.

	Vessel 1	Vessel 2	Vessel 3	Firm
Revenue without cooperation	22,400.0	16,800.0	16,800.0	293,237.2
Revenue with cooperation	40,659.9	36,453.9	35,607.9	327,245.8
Total improvement cooperation	18,259.9	19,653.9	18,807.9	34,008.6
Improvement cooperation (%)	82%	117%	112%	12%

Table 5 but also confirms the theoretical results presented in the previous section.

Finally, in addition to assessing the final number of FADs, according to the theoretical propositions, it must be taken into account that the contribution of each vessel may be different and is determined by the quality of the FADs shared (i.e., through the tons of tuna they manage to attract). Depending on each skipper’s expertise, certain FADs may be better localized (“harvested” in the jargon) due to various factors, such as oceanographic currents, water temperature, surface currents, and so forth. Thus, the amount of tuna caught may be substantially different. To take this into account and avoid further complexities, we have considered two possible situations: the standard FADs, where the expected amount of tuna is six tons on average, and the premium FADs (better “harvested”) where the expected amount of tuna is eight tons on average. Our analysis could easily be extended to the general case, in which FADs could have different tons beyond these two simplifications. According to Proposition 2, the cooperation revenue of each vessel and the firm can be divided into two parts. One part depends on the amount of tuna: for the vessels from their own FADs and for the firm from the total FADs of all its vessels. The other part depends on the distances traveled and, thus, on the savings generated after reassignment of FADs. Assuming that, in the analyzed experiment (Table 4), all the FADs of Vessel 1 are now Premium (instead of Standard) and that all other parameters remain the same, we compare (using Proposition 2) the revenues of all agents (Table 7).

Comparing the results of this table with those of Table 4 (only the last part, the Shapley section), we can observe the following conclusions. First, Vessels 2 and 3 are not affected at all. Second, as the second part (distances) does not change, Vessel 1’s revenue increases by $5600 because the amount of tuna in its FADs is higher, and the firm’s revenue increases. Thus, the total improvement of Vessel 1 and the firm is maintained, but the percentage improvement, with respect to the total, has decreased.

The qualitative conclusions of the analysis could be extended, through Proposition 2, to the general case. Suppose that, in a tuna vessel problem, the FADs of some vessels are premium, and the FADs of other vessels are standard. Thus, the vessels’ revenues with premium FADs and those of the firm would increase (relative to the case where all FADs are standard), and the revenues of the vessels with standard FADs would remain the same. We believe that this monotonicity of Shapley value revenues in this context is an important property. As we have argued previously, in general cooperative games, it could be the case that the improved performance of some agent produces a negative externality on the rest of the players. On the other hand, these results help with the implementation of this type of solution in the industry because teamwork is rewarded. Those who best sow their FADs are rewarded, so this type of initiative, in addition to being beneficial for all entrepreneurs, should be promoted by firms, as it “brings out the best in each agent” resulting in greater benefits for all.

6. Concluding remarks

In this research, we have considered the problem of tuna vessels and the possibility of cooperation among them to maximize their own and the owner firm’s profits. The firm bears the overhead costs (for which fuel oil is by far the main one), and the vessels’ skippers keep a portion of the catches, hence the possible mutual interest to cooperate. When FADs are shared among vessels, fuel consumption is reduced, which generates additional revenue. Our goal is to propose a fair way of sharing the total revenue between the vessels and the firm that also incentivizes this type of initiative.

With this idea in mind, we have modeled the tuna vessels problem as a cooperative game, studying the Shapley value. The theoretical analysis has allowed us to find three theoretical results with the Shapley value. First, the Shapley value provides each agent (the vessels and the firm) with at least the same revenue as when the agents do not cooperate. Second, we have decomposed the Shapley value into two subparts. On the one hand, a part depends exclusively on the tuna catches (for each vessel, its FADs, and for the firm, the FADs of all of its vessels), and another depends on the fuel savings due to the decrease in the distance traveled. Third, we have found that, when there are two vessels, the additional revenue generated by the cooperation is divided equally among the three agents (the two vessels and the firm).

From an empirical point of view, we have calculated the Shapley value for several real-life examples. The conclusions obtained as a result of the empirical analysis show that the Shapley value proposes a fairer way of distributing the generated revenues, as it takes into account what each player actually contributes. Our results not only corroborate the conclusions reached by authors such as Groba et al. (2020) but also demonstrate how the Shapley value is a more equitable distribution method for cooperative games.

The results also show a clear improvement for all parties, starting with the vessels, the firm, and the environment. The new FAD sharing strategy, in addition to showing the best economic results, reduces emissions into the atmosphere, making the tuna fishing industry more sustainable. It is important to note that, for a given speed, the distances saved are equivalent to the fuel savings. As stated by Groba et al. (2015), this is important not only to reduce costs but also to increase potential storage space. Therefore, we could conclude that the proposed solution allows fishing vessels, and any other agent that travels to moving targets in the short term, to minimize the distance traveled, which would have a direct impact on variables such as the time spent, fuel consumption, or CO_2 emissions into the atmosphere.

We recognize that other analyses could be conducted in this setting. For example, we can study the game theoretic properties of the cooperative game, such as convexity or balance. Instead of the Shapley value, we can consider other cooperative values, such as the nucleolus or the τ-value. These possibilities open future avenues of research.

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